PRODUCT ENGINEERING

VOLUME 7 NUMBER 7

MASON BRITTON, *Vice-President*

G. F. NORDENHOLT, *Editor* — K. H. CONDIT, *Consulting Editor*

F. A. FIRNHABER, *Assistant Editor* — JOSEPH KERR, *Assistant Editor*

LEONARD CHURCH, *Cleveland* — PAUL WOOTON, *Washington*

W. E. KENNEDY, *Manager*

August Number

As a companion to the article "Vibration Damping Tests for Resilient Mountings," appearing in this issue, in August *P.E.* Dr. H. O. Fuchs of Delco Products Company will present an article "Spiral Damping Diagrams." This article gives an analytical method for determining the effect of any kind of resistance or damping.

"Deflections and Stresses in Coiled Springs Made of Rectangular Wire," by E. W. Stewart of The William Gibson Company is a valuable contribution to literature now available. Equations developed and checked with actual test data.

Establishment of tolerances is usually taken to be largely a matter of judgment based on a knowledge of production capabilities and economical considerations. A scientific method for establishing tolerances for interchangeable manufacture will be explained in August *P.E.* in an article written by W. J. Ettinger of the Edison-G.E. Appliance Company and Dr. Walter Bartky of the University of Chicago.

Supplementing our article on hydraulic operation in the June issue, the Pratt & Whitney hydraulic helical gear grinder will be featured. Both the hydraulic and electrical diagrams with photographs and text will be given.

An article will describe new features of the Westinghouse hermetically sealed condensing unit for air-conditioning, developed after making a study of units now on the market. Improvements in design and manufacturing set-up will be outlined.

Published monthly. Price 35c. a copy. Subscription rates—United States, Canada, Mexico, Central and South American countries $3 a year. All other countries $5 a year or 20 shillings. Entered as second-class matter January 17, 1930, at Post Office at New York, N. Y., under act of March 3, 1879. Printed in U.S.A.

Branch offices: 520 North Michigan Avenue, Chicago; 883 Mission Street, San Francisco; Aldwych House, Aldwych, London, W. C. 2; Washington; Philadelphia; Cleveland; Detroit; St. Louis; Boston; Atlanta, Ga.

CONTENTS for JULY, 1936

VIBRATION DAMPING TESTS FOR MOUNTINGS 246
S. D. SABLOVE—A method for measuring and recording vibrations in three directions simultaneously.

STRESSES AND DEFLECTIONS IN FLAT PLATES 252
RAYMOND J. ROARK—Tabulation of the equations for plates of different shapes, supports and loadings.

B.E.C. MACHINE FOR TESTING GREASES 255
Description of machine with recommended procedure for determining the mechanical stability of lubricating greases.

COLOR TAKES TO THE RAILS 258
HENRY DREYFUSS—Description of color schemes, appointments and coach arrangements of the streamlined train "Mercury."

DRAFTING ROOM METHODS AND MANAGEMENT—II 260
J. K. OLSEN—Concluding article contains suggestions for uniform specifications on drawings, parts lists and revision record sheets.

ANALYSIS BY THE X-RAY DIFFRACTION METHOD—II .. 264
H. R. ISENBURGER—How X-rays show the results of different heat-treatments, strain hardening and similar effects.

HOW TO MAKE ALIGNMENT CHARTS—V 267
M. G. VAN VOORHIS—Introduction of the curved-scale nomogram and the methods of locating and laying out the scales.

ELECTRIC SWITCHES AND SWITCH MOUNTINGS—II 274
B. B. RAMEY—Typical mountings of unit-type switches as applied to portable motor-driven machines. Concluding installment.

Editorial 245
Product Developments 249
Question and Comment 271
News 276
New Materials and Parts 278
Manufacturers' Publications 281
Books and Bulletins 282
Reference Book Sheet 283

Contents Copyrighted, 1936, by McGraw-Hill Publishing Company, Inc.

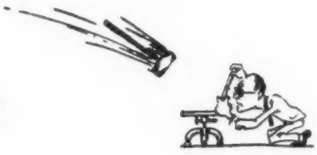
McGRAW-HILL PUBLISHING COMPANY, INC.
330 West 42d Street, New York, N. Y.

JAMES H. McGRAW, JR., Chairman — MALCOLM MUIR, President — JAMES H. McGRAW, Honorary Chairman

MASON BRITTON, Vice-President — B. R. PUTNAM, Treasurer — D. C. McGRAW, Secretary

Cable address: MCGRAWHILL, New York, Member A.B.P. Member A.B.C.

Intimate Correspondence



Enough is Sufficient

PRODUCT ENGINEERING
NEW YORK, N. Y.

Thank you very much for your letter of June 12 with a complete statement of the information desired.

May we add that this is a refreshing experience for inquiries addressed to other editors concerning items referred to in their various publications have been so briefly answered that their replies were almost valueless or at most supplied such meager information that considerable correspondence was involved.

Thanks again for the interested and helpful attention you showed toward our inquiry.

—L.I.L.
Attleboro, Mass.

WE must confess that at times we also are guilty of answering inquiries in a manner that requires further correspondence, but usually we are able to give the data and information to a sufficient extent so that the inquirer will know whether or not it is something that he wishes to follow up in greater detail. The above letter refers to Item (18) that appeared in our "Do You Know That" column in the June number.

But we warn our readers that we cannot act in the capacity of consulting engineers in the solution of detailed problems. In such instances, all that we can do is to refer them to possible sources of information, or refer them to companies that might solve their problems.

Why the Secrecy?

PRODUCT ENGINEERING
NEW YORK, N. Y.

Several times I have been greatly aggravated by getting in touch with some of the letter-writers in *Product Engineering* who signed themselves with initials or a nom-de-plume. I have two objections to disguising the name of the writers of the letters. In the first place, I have to write to you to get his name and address. Secondly, he often turns out to be someone who is merely trying to get some free information, has an ax to grind, or whose interests are in an entirely different field from what one would suspect.

It would help matters a lot if you gave the correct name and address of the individuals whose letters you publish. This would enable direct correspondence and furthermore would identify the field of interest of the writer.

I trust that this suggestion will be given serious consideration. —W.I.C.
Milwaukee, Wis.

WE have given the above suggestion serious consideration on a number of occasions. At one time we did publish the names and addresses but found that in some instances it turned out to be embarrassing for the writer of the letter. In order to avoid that possibility, we resorted to the use of initials. We will be glad to give the name and address of the writer but only in such instances wherein he specifically gives permission to publish it.

More on Appliances

PRODUCT ENGINEERING
NEW YORK, N. Y.

I have subscribed to *Product Engineering* almost from the first issue and consider it more valuable than a textbook, being constantly up-to-date, and often even ahead of the times. I would like to see more articles and data on electrical home appliances, principally because I help design them.

—L.B.R.
Rochester, N. Y.

WE confess that the number of we have had relating to electrical home appliances have been relatively few. The most recent was the article "Strain Reliefs and Protectors" which appeared in our March number. Another article on appliance devices is in the course of preparation. We will be glad to receive further suggestions.

Book Wanted

PRODUCT ENGINEERING
NEW YORK, N. Y.

I am interested in obtaining a copy of "Production Design" by J. K. Olsen, formerly chief draftsman of Stewart-Warner Corporation, which book I understand is out of print. Could you help me in this regard? —L.C.S.
Racine, Wis.

THIS book, by J. K. Olsen is an old one and it is out of print, the publisher having no more copies available. If any of our readers know where a copy is obtainable, please let us know, so that we can pass along the information.

Metal Design

PRODUCT ENGINEERING
NEW YORK, N. Y.

I was extremely interested in the article on page 193 of the May number concerning courses in Pressed Metal Design. Could you ascertain if there is any possibility of this subject being offered as a correspondence or extension course, to make it available to others not living in the cities mentioned? I am sure many others besides myself, among your readers, would be interested. —J.L.B.
Syracuse, N. Y.

The course referred to is given by the University of Toledo, with E. V. Crane of the engineering staffs of the Toledo Machine & Tool Company, and the E. W. Bliss Company giving the course. Full details regarding the possibility of taking the course by correspondence or extension can be obtained by addressing Professor Zarobski of the mechanical engineering department of the University of Toledo, Toledo, Ohio. Perhaps if a sufficient number were interested in such a correspondence course, it might be made available.

Books on Diesels

PRODUCT ENGINEERING
NEW YORK, N. Y.

I want to tell you how much I appreciate your magazine. It is easily the best in my opinion for mechanical engineers. I find it extremely interesting and useful.

At the present moment I am in need of information on "Waste Heat Utilization" in connection with diesel engines. That is, using the exhaust gases to heat water and generate it into steam. I will be glad if you can supply any information or let me know of the most modern book on diesels.

—G.H.B.
Cleveland, Ohio

AMONG the latest books on diesels are "Diesel Engines" by Anderson, McGraw-Hill Book Co., 330 W. 42d St., New York, N. Y.; "Modern Diesel Engine Practice" by Adams, and "Elements of Diesel Engineering" by Adams. Both of these last two are published by Norman H. Henley Publishing Co., 2 West 45th St., New York, N. Y. The last of the three books mentioned deals more with the mechanical design of the engines than do the first two books.

However, with reference to waste heat utilization, the basis of this is the thermodynamics of the engine, rather than its mechanical design.

Rubber Standards

PRODUCT ENGINEERING
NEW YORK, N. Y.

I would like to secure information regarding methods and standards most widely used for testing rubber. Of particular interest are the methods used to check the durability and grading of either molded or strip rubber.

Any information you can forward to the attention of the writer will be greatly appreciated. —J.O.
Buffalo, N. Y.

THE American Society for Testing Materials, 260 S. Broad St., Philadelphia, Pa., issues standards and tentative standards covering specifications and tests for rubber products. Copies of such standards can be purchased at a nominal price. And in view of the fact that these standards and tests have been endorsed by both manufacturers and users, they are the most authoritative available.

PRODUCT ENGINEERING

GEORGE F. NORDENHOLT, *Editor*

MCGRAW-HILL PUBLISHING CO., INC. JULY 1936 VOLUME SEVEN, NUMBER SEVEN

Science and Social Progress

RESEARCH and scientific investigation are so obviously fundamental to the work of the designing engineer that they need no defense to him. His principal interest is that they be supported and stimulated as much as possible so that he may turn their discoveries to practical use. In so doing he makes of them a powerful aid to social progress—through new industries that give new opportunities for employment, new facilities and conveniences for the consumer, new fields of investment for capital.

In this country, science functions with the support of the government, of private industry, and of the colleges and universities. President Compton of Massachusetts Institute of Technology, who served as chairman of the Science Advisory Board until it ended its career in December, says of these branches of science that each faces serious handicaps that could and should be removed.

Scientific bureaus of the government are sadly in need of coordination. The scientists connected with them are able and conscientious but they suffer from bureaucratic rivalries, and from the uncertain tenure of politically appointed chiefs. They start many projects, stop very few. They duplicate effort through lack of knowledge of what is being done in other bureaus. They get scant encouragement from Congress and the Executive, busy with political questions. Yet the government needs their dispassionate, unbiased opinion on many technical matters.

A permanent scientific advisory council, unpaid, was proposed by the temporary Board to coordinate the scientific work of the bureaus, to defend them before Congressional committees, and to recommend proper budgets for their projects. Several European countries are far ahead of us in this respect. In England, for instance, almost exactly the set-up suggested for the United States is giving invaluable service to the privy council.

Where government research may well concern itself with fundamental problems, industrial research can properly get down to more detailed questions. Even so, the indirect benefits of its discoveries may easily be several times more profitable than the information which was directly sought and obtained.

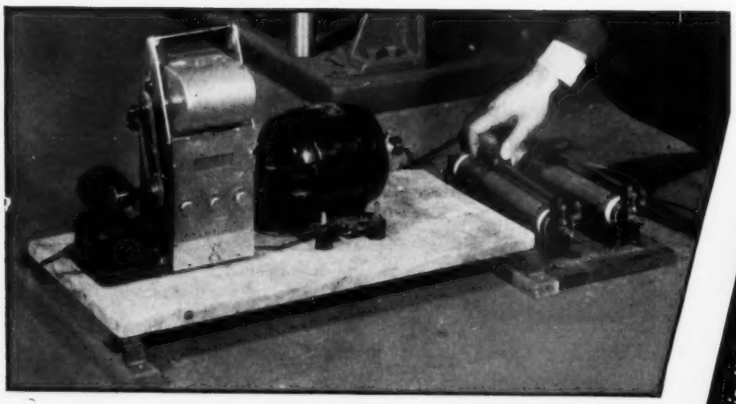
But industrial research has its obstacles, too. There is the "practical" type of business man who lacks the vision to see what research can do, and the financial management that is equally effective in killing a business. There is our outmoded patent procedure, and there is the increasing burden of government regulation. Government should help instead of hinder.

In the university laboratories, however, lies the source of most of the new ideas which industry, through its product engineers and scientific staffs, converts to the furthering of social progress. The need here is for more financial support and closer contact with industry. The average designing engineer is not in a position to endow academic research, but he can do much to improve the contacts between industry and the schools. He has classmates on the teaching staffs, and he does, or should, go back occasionally to his old school to see what is going on and keep in touch with faculty friends. If, in addition, he will make an effort to follow developments at other educational institutions, through visits or through attendance at meetings of engineering societies, he can do much to foster this very valuable cooperation.

Vibration Damping Tests for Resilient Mountings

Equations for critical speeds with elastic suspensions, and the vibrograph method for selecting best type of rubber suspension

S. D. SABLOVE
The Drexel Institute



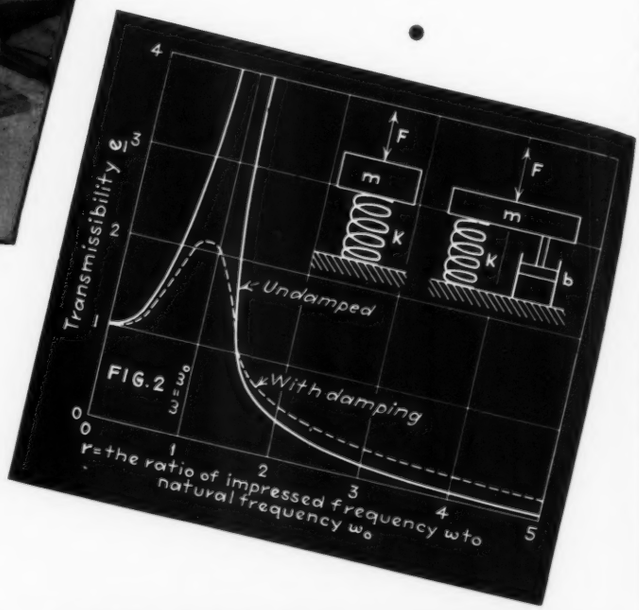


Fig. 1—Apparatus used for investigating rubber mountings. Fig. 2—Transmissibility of simple vibrating systems with and without damping. Fig. 3—Transmissibility for a system taking into account the elasticity of the foundation

WITH the present-day trend toward larger machines and increased speeds the problem of vibration has become one of extreme importance, not only in the balancing of rotating machinery, but in the isolation of objectionable vibrations. Elastic suspension systems using springs, shear rubber, cork or air chambers are being used to prevent the transmission of machine vibrations to floors or foundations.

The effectiveness of an elastic suspension is specified by its transmissibility, or the ratio of the disturbance produced in the foundation with a suspension to that produced with a rigid mounting.

In using the method of mechanical impedance to study vibration, consider a system of mass m supported by a spring having a deflection constant K upon which a periodic force of amplitude F and angular velocity ω is impressed, as shown in the diagram in Fig. 2. Then the displacement y is given by

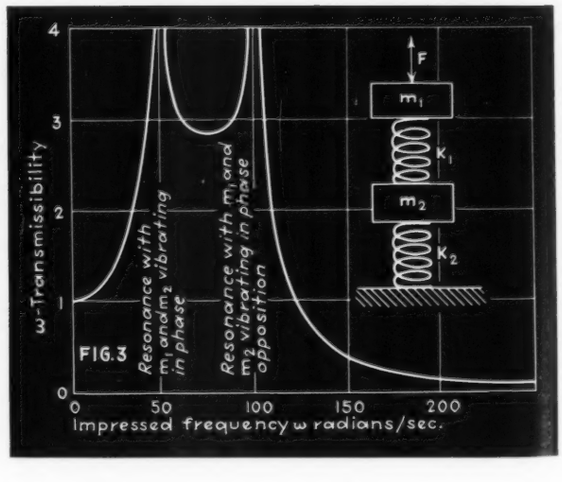
$$y = F/(K - m\omega^2)$$

The force transmitted to the foundation through the spring is

$$K_y = KF/(K - m\omega^2)$$

If the spring is replaced by a rigid support, the force transmitted to the foundation is simply F. Therefore, the transmissibility e is

$$e = \frac{K}{m} \Big/ \left(\frac{K}{m} - \omega^2\right)$$



The natural frequency ω_0 in radians per sec. of the mass on the spring is $\sqrt{K/m}$. Therefore

$$e = \frac{1}{1 - (\omega^2/\omega_0^2)}$$

For convenience the ratio of ω/ω_0 is designated r, and

$$e = 1/(1 - r^2)$$

The relation between e and r is shown graphically in Fig. 2. The transmissibility e is considered a positive quantity since the negative sign merely indicates phase opposition between the two forces.

If $\omega = \omega_0$, a resonant condition is reached, and the transmissibility becomes very large. For any reduction in transmissibility, ω_0 must be less than ω. For a satisfactory isolation system r must be about 6 or 7 so that e becomes

$$e = 1/(1 - 7^2) = 1/48 = 0.021$$

By the addition of a dash-pot in parallel with the spring a damping factor b proportional to the velocity is introduced in this simple system, shown in diagram, Fig. 2. The transmissibility in terms of the impressed frequency f and natural frequency f_0 of the supported mass, both frequencies being expressed in cycles per sec., is given by the equation:

$$e = \sqrt{(f_0^2 + B)/[(f_0^2 - f^2)^2 + B]}$$

In this relation, the term B is equal to $b^2f^2/4\pi^2m^2$. When b is zero this equation reduces to that of the simple case without damping. With damping present the transmissibility curve for this system is similar to the first system except that a decrease in transmissibility is obtained at the resonant frequency. As shown by the curves in Fig. 2, the damping is effective in the critical range, but is slightly detrimental at the higher speeds. Thus, under certain conditions of operation damping may slightly increase the vibration if it is improperly applied.

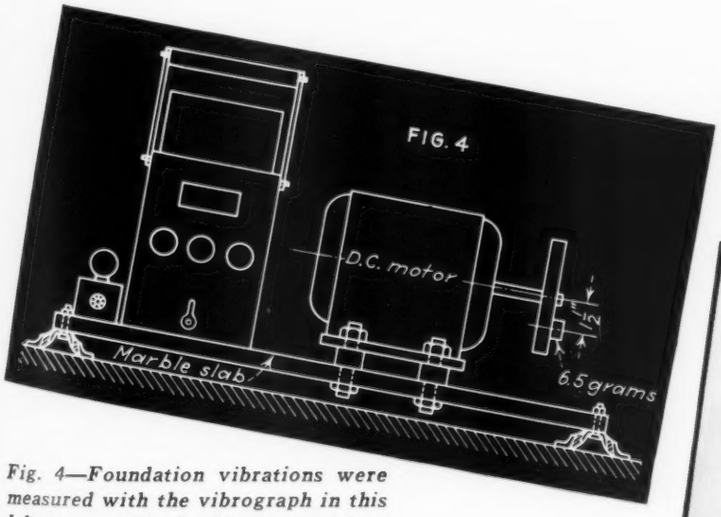


Fig. 4—Foundation vibrations were measured with the vibrograph in this laboratory test of rubber suspensions

Laboratory Test Method

Systems usually encountered in practice are much more complicated than the simple systems illustrated in Fig. 1. A perfectly rigid foundation does not exist. The usual case, neglecting damping, approximates more closely the system shown in Fig. 3, which includes mass m_2 and the elastic constant K_2 of the foundation itself. This system has two resonant frequencies given by

$$= 0.113\sqrt{A \pm \sqrt{A^2 - \frac{4K_1K_2}{m_1m_2}}}$$

$$A = \frac{K_1}{m_2} + \frac{K_2}{m_2} + \frac{K_1}{m_1}$$

At the lower of the two resonant frequencies the two weights vibrate in phase while at the higher frequency they vibrate in phase opposition. Fig. 3 shows the transmissibility curve for such a system.

Since such a complex system does exist in usual practice the set-up of the apparatus for an investigation of suspension systems was made to approximate the real condition. In determining the performance of elastic rubber suspensions the test set-up shown in Fig. 4 was constructed. A d.c. motor supplied the harmonic force

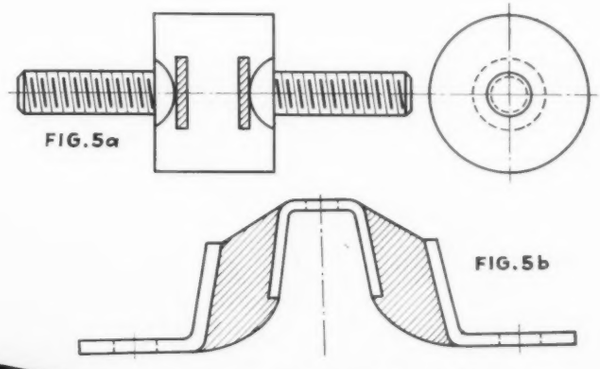


Fig. 5—Compression rubber mounting (a) and the shear rubber type (b) used in the tests

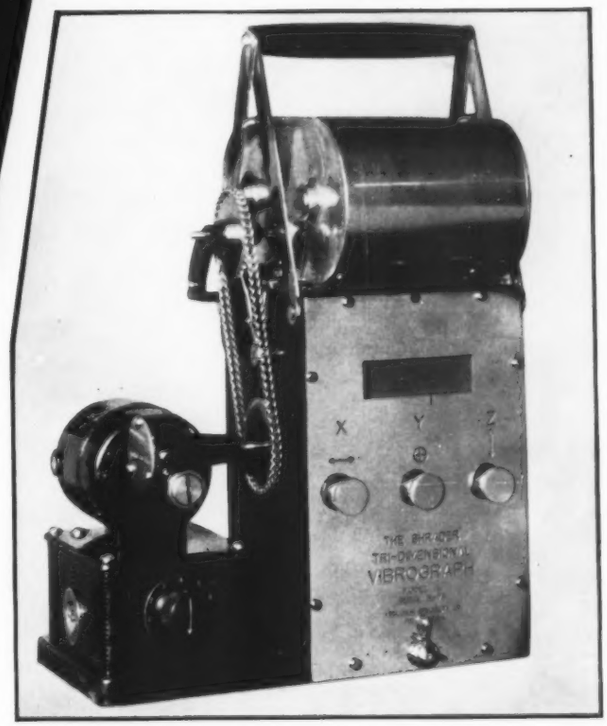


Fig. 6—Tri-dimensional vibrograph used to measure amplitude, phase and frequency of vibrations in three coordinate planes

on the system by a 6.5 gram weight placed eccentrically on the pulley at a distance of 1.5 in. from the shaft center. The motor was isolated from a marble slab by four Firestone cushion connectors of the type shown in Fig. 5*a*. The marble slab was in turn supported at its four corners by Firestone vibration dampeners, Fig. 5*b*, which rested on a concrete foundation. The vibration dampeners were designed so that the load was taken by rubber in shear.

With the motor running at various speeds, the vibrations of the marble slab were measured by the use of a tri-dimensional vibrograph developed by Dr. J. E. Shrader of Drexel Institute. This vibrograph shown in Fig. 6 is a form of seismograph which records photographically on a moving film, components of vibrations in three dimensions simultaneously. The record shows amplitude, wave form, phase relation and frequency of the three components of vibration. The motor speed as well as the frequency of the component vibrations were obtained by means of a timing device built into the instrument. A rotating mirror also provided for visual observation of the three vibrations so that critical points were easily located.

All tests were made with the marble slab supported on rubber, but various supports for the unbalanced motor were tried. In each test the vibrations were recorded in the top curve for vertical components, in the middle curve for the crosswise or horizontal components at right angles to the shaft, and in the bottom curve for lengthwise components in the direction of the shaft.

With the motor rigidly connected to the marble slab, a resonance condition is reached at 1,215 r.p.m. in the vertical direction, as shown in Fig. 7. With further increase in speed up to 1,850 r.p.m. the amplitude of this vibration is considerably decreased and the high frequency components are almost completely damped out. Both of the other components are irregular in wave shape, as the result of a rocking motion of the system, and end play of the motor shaft.

When the motor was isolated from the marble base by the No. 40 cushion connector, a critical speed in the vertical direction was found at 1,310 r.p.m., as shown in Fig. 8. Amplitudes of the crosswise components were large in the range of 1,200 to 1,390 r.p.m. At the higher speed of 1,850 r.p.m. the vertical vibration reduced to small ripples, while the other two components were favorably damped.

Under the No. 50 cushion connector, Fig. 9, large vibrations were obtained in all directions at 1,275 r.p.m., while at the lower speed of 930 r.p.m. the crosswise component was critical, indicating a small amount of damping in that direction. It will be noted that the peaks of the waves that would be expected to be in line follow a low period of about one-fourth of the impressed wave frequency. This results from rocking of the system about the vertical axis. At the highest speed of 1,810 r.p.m. the vibrations are considerably reduced.

With the No. 60 connector the vibrations in the crosswise direction increased with speed and reached a maximum value at 1,860 r.p.m. the highest speed obtainable with the motor. As shown in Fig. 10, both the other vibrations were well damped out at this high speed.

The records obtained from these tests give a good indication of the transmissibility of such a system at different speeds and show the ease with which vibrations may be analyzed. In this investigation no attempt was made to choose the proper size of elastic materials to damp out the vibrations of the system, and elastic constants and damping factors were not calculated. However, the records show that proper selection of materials is necessary to make the suspension system effective.

In dealing with rubber it must be remembered that it is a compounded material that has been given some form or other of heat-treatment. Hence different types of rubber will have widely varying damping characteristics.

This article, submitted for exclusive publication in Product Engineering, *is based on a thesis study sponsored by Firestone Tire & Rubber Company at The Drexel Institute, directed by Dr. J. E. Shrader.*

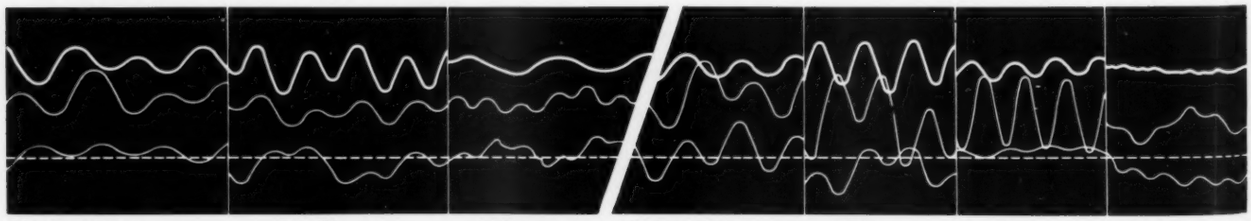
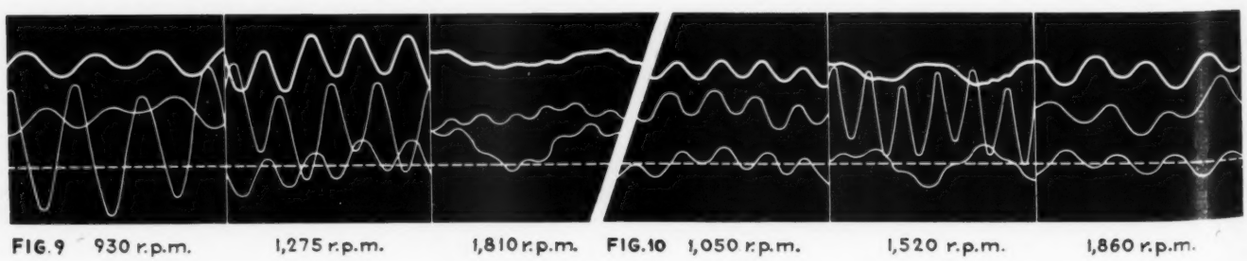


Fig. 7—Foundation vibrations at three speeds with the motor rigidly attached to the base. Fig. 8—Vibrations found with the motor isolated with No. 40 Firestone cushion connectors. Fig. 9—With No. 50 cushion connectors. Fig. 10—With No. 60 cushion connectors



PRODUCT DEVELOPMENTS

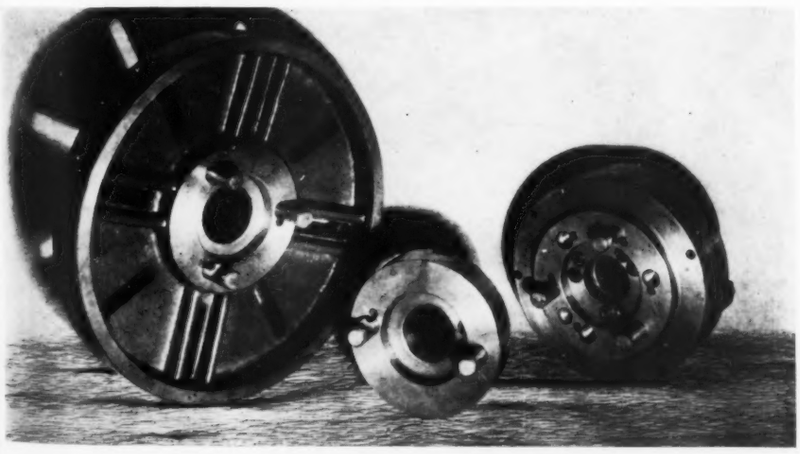
Cam-Lock Flanged Spindle for Face Plates and Chucks

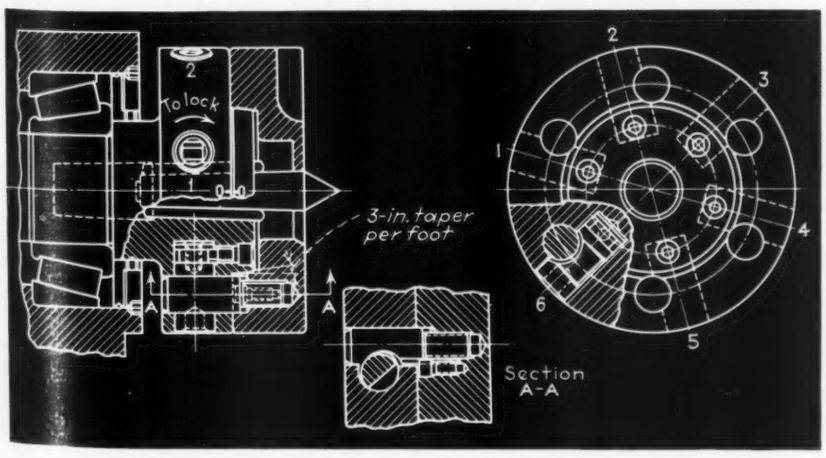
● Face plates, chucks and driver plates are attached to the Monarch lathe spindle with a cam type of clamp. The so-called cam-lock spindle nose is an integral part of the headstock spindle and is arranged with six cams that lock the cam studs of the attached member. The hardened cams are retained by socket head cap screws. A square end T-wrench engages the ends of these cams, a quarter turn being sufficient to lock the attached member in place.

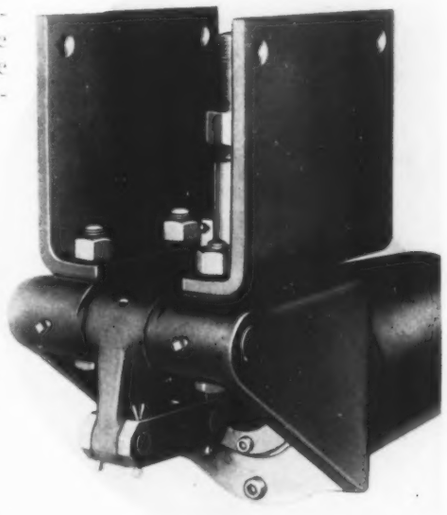
The hardened cam studs are threaded into the chuck or plate and fastened with socket head screws. These studs serve both as drivers and to hold the parts in place while the operator clamps them in position. The attached member is centered on the tapered portion of the extreme front end of the spindle, having a 3-in. per ft. taper. The entire spindle is hardened to approximately 60 Rockwell C-scale.

To make interchangeability possible, the same size of spindle nose is used on all lathes of a given range of sizes. This includes engine, turret and automatic types of lathes. The driver plate has two studs, and the face plate and chuck have three studs each, fitting in the numbered holes in the spindle as specified. The advantage of this cam-lock type of spindle lies principally in the quickness with which the chucks, plates or fixtures can be clamped to the spindle nose or removed. Another feature is that the attached member is held securely and cannot work loose even under the most severe conditions.

Cam studs in face plate, chuck and driver plate fit directly into corresponding holes in spindle nose. Cams engage studs, securely clamping fixture to tapered spindle nose



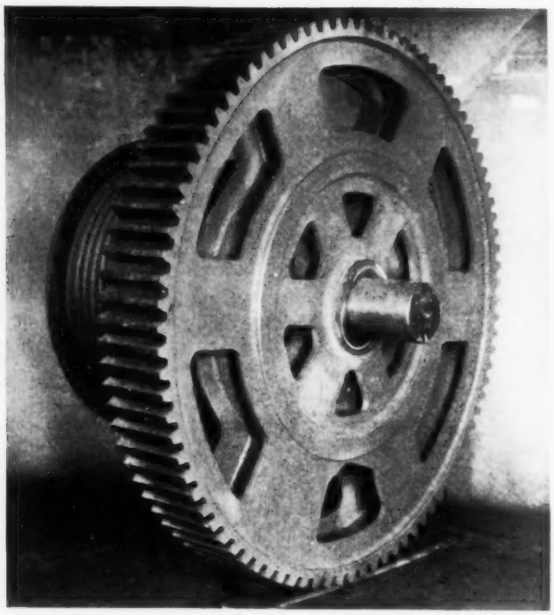




Electrically-controlled air operator for manipulating mechanical friction clutches and brakes on large power presses

● *Air-Electric Control*

To eliminate manual operation of mechanical friction clutches and brakes on large power presses, requiring several operators, an electrically-controlled, air-operated device has been developed by the Cleveland Punch & Shear Works Company. It is operated by control buttons either by hand or foot and so arranged that only when all the buttons are depressed will the clutch be thrown into engagement. It is unnecessary for one operator to wait for another before pressing the controls assigned to him. However,



Double spoked all-welded steel gears made for use on cranes by the Harnischfeger Corporation

Below to the right is the new electric fan designed by Robert Heller for A. C. Gilbert Company. The maroon colored motor with black fan blades is supported by a chrome guard, which in turn is fastened to the airplane wing base. The former design is at the left

any individual operator can stop the press at any part of its stroke.

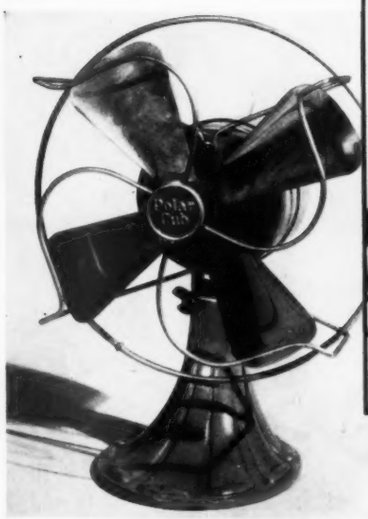
In the electrical equipment there is a 4-position selector for momentary, long-stroke, inching and continuous operation.

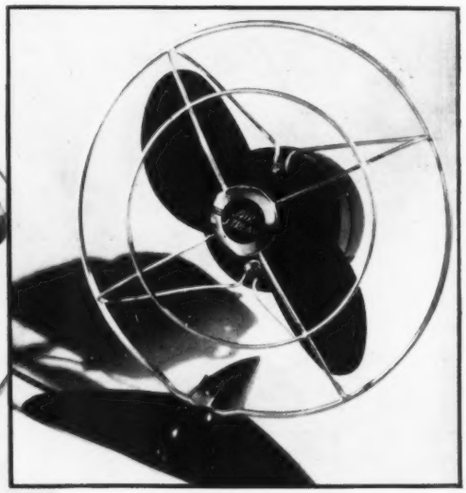
Greater sensitivity is obtained by virtue of the fact that the clutch can be engaged and disengaged rapidly. This eliminates dragging and consequent wear of the clutch surfaces. Because there is more power available to operate the clutch and brake than when thrown by hand, the surfaces can be set to less clearance. The device is adjustable whereby drag and resultant heating and wear are eliminated.

a greater resistance to wear and strain.

The gears are built up of arms or webs of rolled steel, either single or double, and reinforced with gear bands welded integrally. When a single set of arms is used, each arm is reinforced with a rib of steel on each side of the arm. When double arms are used they may or may not be ribbed. The band or ring into which the teeth are cut is of an alloy steel selected for the service. It is welded together at the joint.

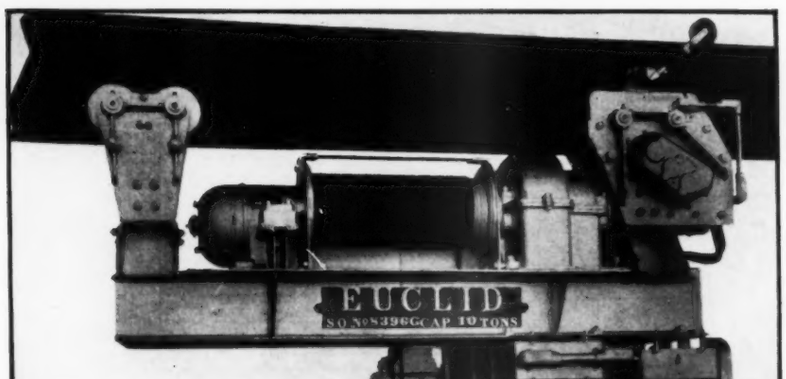
After welding, the entire unit is normalized to provide the greatest possible strength. The gear is then machined, the teeth are cut, and heat-treated to suit.





• *Aerodynamics in Electric Fan Design*

The design of this new type of fan is a departure from the former type in that it has a two-blade fan instead of four blades. The inspiration was derived from the airplane, the design being based on aerodynamic principles. It was found that the four-bladed type was too closely spaced to give an efficient air blast.



Lincoln Electric Company

Steel fabrication of a 10-ton floor-controlled monorail hoist eliminates expense of drawings, patterns, molding and machining of castings

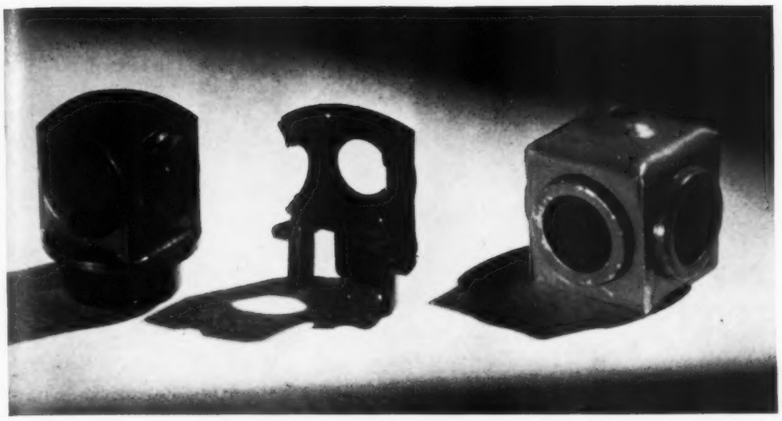
• *Welded Crane Gears*

Crane gears are now fabricated by the Harnischfeger Corporation out of rolled steel through the use of welding. Besides the reduction of weight, the gears are said to have

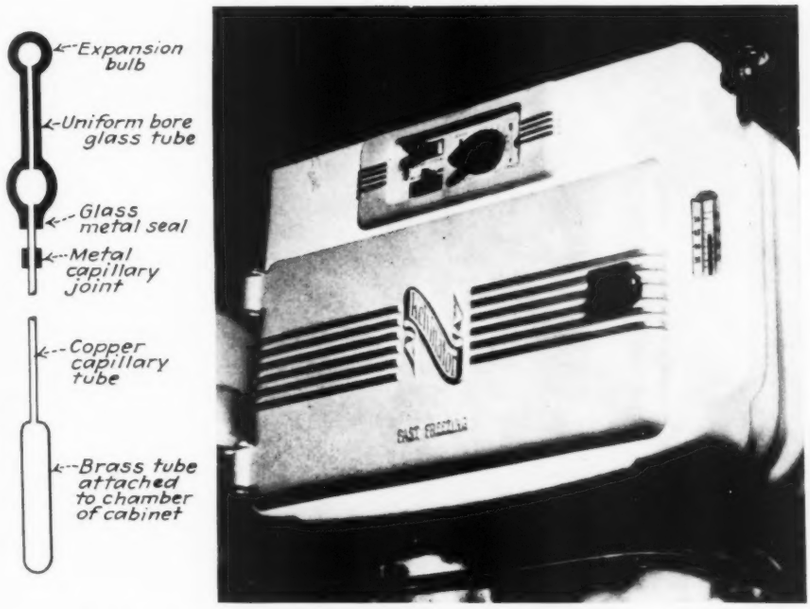
• *Monorail Hoist Is All-Steel Welded*

No cast iron or steel castings are used in the design of the monorail hoist built for the United States Air Depot at Middletown, Pa., by the Euclid Crane & Hoist Company. Standard I-beams, channels, angles and plates, cut to proper sizes, are arc-welded together.

Main frame members are of 25.4-



Toy railway signal lamp housing is extrusion-molded of glossy-finished black Tenite



The "Visible Cold" built-in thermometer in the Kelvinator gives the user an opportunity to set the temperature controls by readings in deg. F. instead of a dial setting

lb. I-beams. The bed plate, frame gussets, vertical and rear king pin supports are fabricated of $\frac{3}{8}$-in. plate and the supports for the front king pin and drum shroud are made of $\frac{1}{4}$-in. plate. The end drum shaft support is a 9.8-lb. channel. The main frame beams are 8 ft. long, the bed plate being 6 ft. 5 in. long by 3 ft. 8 in. wide.

In the arc-welded, steel-driving trolley, not only was the cost of material considerably reduced but the weight was also reduced from 1,519 lb. to 728 lb., a saving of 51½ per cent over the steel casting type of trolley.

Although both the old and new types of hoist are of the same capacity, the welded steel unit is only two thirds as large as the former type

• *Extrusion Molded Lamp Housing*

Whereas a metal stamping was formerly used for the lamp housing of a toy railway signal switch made by the Lionel Corporation, a molded cellulose acetate part is now used. The molded housing was made by the Boonton Molding Company in one operation in an extrusion molding machine as compared to eight stamping operations on the metal part. The latter had a main housing, four small rings and four disks of celluloid for the windows.

The plastic piece, molded cornerwise, has a small radial fin at each corner of the interior. Between these fins are slipped four rectangular pieces of celluloid for the windows which are held in with a touch of acetate. The piece is then completed. In the superseded part it was difficult to quickly fasten the round window disks. This part also necessitated painting, whereas the molded part has a glossy and permanent black finish.

• *Thermometer Built-In*

To afford the user a check of the temperature in an electric refrigerator, the Kelvinator Corporation has included a built-in thermometer. This indicator is attached to the front cooling unit baffle and is calibrated from 35 to 50 deg. F. to give average compartment temperature. The bulb is attached to the cabinet chamber wall to prevent sudden rise in indicated temperatures when the door is opened. Operation depends on expansion or contraction of Meta Zylene which has a high coefficient of expansion.

• *Random Jottings About New Developments*

Four sets of radius gages have been recently announced by Lufkin Rule Company, each gage having five applications of the same size radius, for both external and internal work.

• • •

Furniture by George J. Pike Company is now being made of "Steelwood," a material with a steel base and a thin layer of wood veneer. The wood is applied to the steel core under heavy pressure. The finished product has the appearance of wood and is said to be extremely strong and will not chip.

• • •

A bench-type power jigsaw, operated by a self-contained magnetic power unit, produces 7,200 cutting strokes per min. It is made by Syncro-Devices, Inc. Used for delicate scroll work on wood, thin metals, veneers and other materials. Runs on 60 cycles, a.c., and is only 19 lb. in weight.

• • •

A new automobile-brake hydraulic cylinder by Weatherhead is made entirely of steel tubing, sheet metal stampings and screw machine parts, all brazed together. The manufacturers state that the cast cylinders used previously developed leakages.

Stresses and Deflections in Flat Plates

RAYMOND J. ROARK
Professor of Mechanics, University of Wisconsin

UNLESS otherwise indicated, all equations in the following tables represent standard formulas based on mathematical analysis. Certain of these equations are admittedly approximate. For this reason some of the equations were verified experimentally. Both cast iron specimens and plaster models were tested, giving the confirmation to the formulas that only tesing can supply.

The formulas given here are based on the following assumptions:

1. The thickness of the plate is uniform, and not more than 0.3 times the least transverse dimension.
2. The maximum deflection of the plate is not more than 0.2 times the thickness.
3. Neither the loads nor the supporting reactions have components in the plane of the plate, hence the plane midway between the top and bottom surface is unstressed.
4. The maximum unit stress does not exceed the limit of proportionality.
5. The material is homogeneous and has the same modulus of elasticity in tension as in compression.
6. For convenience in referring to directions, the plate is assumed to be horizontal and loads and reactions vertical.

Notations

W = total applied load in lb.
w = unit applied load in lb. per sq.in.
q = any given point on the surface of the plate
r = distance in in. of q from center of circular plates
t = thickness of the plate in in.
S = unit stress at surface of plate in lb. per sq.in.
$+$ = indicates tension in upper surface and equal compression in lower surface
$-$ = indicates compression in upper surface and equal tension in lower surface
Subscripts r, t, a and b used with S denote respectively radial direction, tangential direction, direction of dimension a, direction of dimension b.
y = vertical deflection of plate from original position, in in., negative sign indicates downward, positive sign indicates upward
E = modulus of elasticity in lb. per sq.in.
M = reciprocal of Poisson's ratio
All logarithms are to the base e.
$\log_e = 2.3026 \log_{10}$

Conditions of support: Complete edge constraint—is seldom attained, and is closely approximated only when the plate is firmly attached to or held between rigid supporting elements. In the majority of actual cases, constraint is partial, and the degree of fixity must be estimated from the relative rigidity of the plate and the supports.

Effect of large deflection: The formulas assume that the load is carried wholly by flexural resistance, and that the plate does not act like a diaphragm and carry part of the load by tension. This assumption is very nearly true when the deflection does not exceed about one-fifth the thickness of the plate, but if the deflection becomes much greater than this, as might be the case with a thin, flexible plate, the effect of tension in the plane of the plate can no longer be ignored. Such a thin plate, if simply supported at the edges, will fail by buckling due to circumferential compression set up by the radial tension. If the edges are held by horizontal radial forces, the resistance of a very thin plate is limited almost wholly by its tensile strength and extensibility; it acts like a diaphragm of fabric or rubber, and its flexural resistance is comparatively negligible.

Concentrated loading: The formulas for maximum stress produced by a load distributed over a small area or along the perimeter of a small ring, as in Cases 2 and 3, give very high results when the radius of the loaded area or ring is small and indicate that the stress approaches infinity as this radius approaches zero. The formulas, however, cannot be considered valid for highly concentrated loading, because they take into account neither the mitigating effect of plastic yielding nor the effect of the stress caused by the direct pressure of the load. It is probable that they have little practical significance when the loaded area is so small that the unit pressure thereon equals or exceeds the elastic limit in bearing. This limitation, however, does not apply to formulas for stress at points remote from the load, nor to formulas for deflection.

Breaking strength: When the formulas for elastic stress are used for predicting breaking strength, the results obtained are too low. The load actually required to break a plate is usually much larger than the value of W found by substituting for S, in the appropriate formula, the ultimate strength of the material, or even the modulus of rupture as determined from tests on rectangular beams. The discrepancy is greater when the highly stressed zone is small than when it is large, and is greater for relatively plastic than for extremely brittle materials. Tests on plaster of Paris plates gave ratios of actual to predicted breaking strength ranging from 1.4 (Case 14, with b/a small) to 0.8 (Case 3, with b/a large). For Case 1 the ratio was 1.1. For cast iron (less brittle) the corresponding ratios were from 10 to 40 per cent greater.

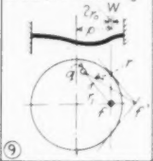
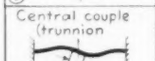
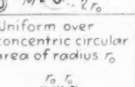
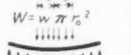
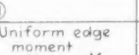
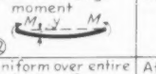
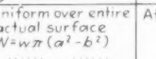
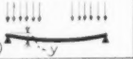
Cases of combined loading: Cases may be encountered which represent a combination of two or more of the loadings given in the tables. They may be solved by applying the Principle of Superposition, according to which principle the stress or deflection resulting from any combination of loads acting simultaneously is equal to the algebraic sum of the stresses or deflections that the several loads would cause if separately applied. This principle is especially useful in calculating the effect of partial edge constraint, of stay-bolt support and similar problems.

Stresses and Deflections in Flat Plates

RAYMOND J. ROARK

Circumstances and Reference Number			STRESS	DEFLECTION
Form of Plate	Manner of Support	Manner of Loading and Reference No.		
Circular and Solid	Edges Supported	Uniform over entire surface $W = w\pi a^2$ ①	At q $S_r = -\frac{3W}{8\pi mt^2}\left[(3m+1)\left(1-\frac{r^2}{a^2}\right)\right]$ $S_t = -\frac{3W}{8\pi mt^2}\left[(3m+1)-(m+3)\frac{r^2}{a^2}\right]$ At center Max $S_r = S_t = -\frac{3W}{8\pi mt^2}(3m+1)$	At q $y = -\frac{3W(m^2-1)}{8\pi Em^2t^3}\left[\frac{(5m+1)a^2}{(m+1)2} + \frac{r^4}{2a^2} - \frac{(3m+1)r^2}{m+1}\right]$ At center Max $y = -\frac{3W(m-1)(5m+1)a^2}{16\pi Em^2t^3}$ At edge Max slope $= \frac{3W(m-1)a}{2\pi Emt^3}$
		Uniform over concentric circular area of radius r_o $W = w\pi r_o^2$ ②	At $q, r < r_o$ $S_r = -\frac{3W}{2\pi mt^2}\left[m+(m+1)\log\frac{a}{r_o} - (m-1)\frac{r_o^2}{4a^2}(3m+1)\frac{r^2}{4r_o^2}\right]$; $S_t = -\frac{3W}{2\pi mt^2}\left[m+(m+1)\log\frac{a}{r_o} - (m-1)\frac{r_o^2}{4a^2} - (m+3)\frac{r^2}{4r_o^2}\right]$ At $q, r > r_o$ $S_r = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r} - (m-1)\frac{r_o^2}{4a^2} + (m-1)\frac{r_o^2}{4r^2}\right]$; $S_t = -\frac{3W}{2\pi mt^2}\left[(m-1)+(m+1)\log\frac{a}{r} - (m-1)\frac{r_o^2}{4a^2} - (m-1)\frac{r_o^2}{4r^2}\right]$ At center Max $S_r = S_t = -\frac{3W}{2\pi mt^2}\left[m+(m+1)\log\frac{a}{r_o} - (m-1)\frac{r_o^2}{4a^2}\right]$	At $q, r < r_o$ $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[4a^2 - 5r_o^2 + \frac{r^4}{r_o^2} - (8r^2+4r_o^2)\log\frac{a}{r_o} - \frac{2(m-1)r_o^2(a^2-r^2)}{(m+1)a^2} + \frac{8m(a^2-r^2)}{m+1}\right]$ At $q, r > r_o$ $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[\frac{(12m+4)(a^2-r^2)}{m+1} - \frac{2(m-1)r_o^2(a^2-r^2)}{(m+1)a^2} - (8r^2+4r_o^2)\log\frac{a}{r}\right]$ For r_o very small (concentrated load) Max $y = -\frac{3W(m-1)(3m+1)a^2}{4\pi Em^2t^3}$ At center Max $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[\frac{(12m+4)a^2}{m+1} - 4r_o^2\log\frac{a}{r_o} - \frac{(7m+3)r_o^2}{m+1}\right]$ At edge Max slope $= \frac{3W(m-1)a}{\pi Emt^3}$
		Uniform on concentric circular ring of radius r_o ③	At $q, r < r_o$ Max $S_r = S_t = -\frac{3W}{2\pi mt^2}\left[\frac{1}{2}(m-1)+(m+1)\log\frac{a}{r_o} - (m-1)\frac{r_o^2}{2a^2}\right]$ At $q, r > r_o$ $S_r = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r} + (m-1)\frac{r_o^2}{2r^2} - (m-1)\frac{r_o^2}{2a^2}\right]$ $S_t = -\frac{3W}{2\pi mt^2}\left[(m-1)+(m+1)\log\frac{a}{r} - (m-1)\frac{r_o^2}{2r^2} - (m-1)\frac{r_o^2}{2a^2}\right]$	At $q, r < r_o$ $y = -\frac{3W(m^2-1)}{2\pi Em^2t^3}\left[\frac{(3m+1)(a^2-r^2)}{2(m+1)} - (r^2+r_o^2)\log\frac{a}{r_o} + (r^2-r_o^2) - \frac{(m-1)r_o^2(a^2-r^2)}{2(m+1)a^2}\right]$ At $q, r > r_o$ $y = -\frac{3W(m^2-1)}{2\pi Em^2t^3}\left[\frac{(3m+1)(a^2-r^2)}{2(m+1)} - (r^2+r_o^2)\log\frac{a}{r_o} - \frac{(m-1)r_o^2(a^2-r^2)}{2(m+1)a^2}\right]$ At center Max $y = -\frac{3W(m^2-1)}{2\pi Em^2t^3}\left[\frac{(3m+1)(a^2-r_o^2)}{2(m+1)} - r_o^2\log\frac{a}{r_o}\right]$
		Uniform over small eccentric circular area of radius r_o Load at f ④ $fq = r_1$, $fg = a_1$	At point of load, f Max $S_r = S_t = -\frac{3W}{2\pi mt^2}\left[m+(m+1)\log\frac{a-p}{r_o} - (m-1)\frac{r_o^2}{4(a-p)^2}\right]$ At q, (1) $S_r = (\text{Max. } S_r)\left[\frac{(m+1)\log\frac{a_1}{r_1}}{1+(m+1)\log\frac{a_1}{r_o}}\right]$ (1) $S_t = (\text{Max. } S_t)\left[\frac{(m+1)\log\frac{a_1}{r_1} + (m-1)}{m+(m+1)\log\frac{a_1}{r_o}}\right]$	At q $y = K_0(r_1^3 - b_0ar_1^2 + c_0a^3) + K_1(r_1^4 - b_1ar_1^3 + c_1a^3r_1)\cos\phi + K_2(r_1^4 - b_2ar_1^3 + c_2a^2r_1^2)\cos 2\phi$ where $K_0 = \frac{2(m+1)W(p^3 - b_0ap^2 + c_0a^3)}{9(5m+1)K\pi a^4}$; $K_1 = \frac{2(3m+1)W(p^4 - b_1ap^3 + c_1a^2p^2)}{3(9m+1)K\pi a^6}$; $K_2 = \frac{(4m+1)^2W(p^4 - b_2ap^3 + c_2a^2p^2)}{(9m+1)(5m+1)K\pi a^6}$ where $K = \frac{m^2Et^3}{12(m^2-1)}$ $b_0 = \frac{3(2m+1)}{2(m+1)}$; $b_1 = \frac{3(4m+1)}{2(3m+1)}$; $b_2 = \frac{2(5m+1)}{4m+1}$ $c_0 = \frac{4m+1}{2(m+1)}$; $c_1 = \frac{6m+1}{2(3m+1)}$; $c_2 = \frac{6m+1}{4m+1}$
		Central couple (trunnion loading) ⑤	At $r = r_o$ (1) Max $S_r = \frac{3M}{4\pi t^2 r}\left[1+\left(\frac{m+1}{m}\right)\log\frac{2(a-r_o)}{ka}\right]$ where $k = \frac{0.49a^2}{(r_o+0.7a)^2}$	
	Fixed edges	Uniform over entire surface $W = w\pi a^2$ ⑥	At q $S_r = -\frac{3W}{8\pi mt^2}\left[(3m+1)\frac{r^2}{a^2} - (m+1)\right]$ $S_t = -\frac{3W}{8\pi mt^2}\left[(m+3)\frac{r^2}{a^2} - (m+1)\right]$ At edge Max $S_r = \frac{3W}{4\pi t^2}$; $S_t = \frac{3W}{4\pi mt^2}$ At center $S_r = S_t = -\frac{3W(m+1)}{8\pi mt^2}$	At q $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[\frac{(a^2-r^2)^2}{a^2}\right]$ At center Max $y = -\frac{3W(m^2-1)a^2}{16\pi Em^2t^3}$
		Uniform over concentric circular area of radius r_o $W = w\pi r_o^2$ ⑦	At $q, r < r_o$ $S_r = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r_o} + (m+1)\frac{r^2}{4a^2} - (3m+1)\frac{r^2}{4r_o^2}\right]$ $S_t = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r_o} + (m+1)\frac{r^2}{4a^2} - (m+3)\frac{r^2}{4r_o^2}\right]$ At $q, r > r_o$ $S_r = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r} + (m+1)\frac{r_o^2}{4a^2} + (m-1)\frac{r_o^2}{4r^2} - m\right]$ $S_t = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r} + (m+1)\frac{r_o^2}{4a^2} - (m-1)\frac{r_o^2}{4r^2} - 1\right]$ At edge $S_r = \frac{3W}{2\pi t^2}\left(1-\frac{r_o^2}{2a^2}\right) = \max S_r$ when $r_o > 0.588a$ $S_t = \frac{3W}{2\pi mt^2}\left(1-\frac{r_o^2}{2a^2}\right)$ At center $S_r = S_t = -\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r_o} + (m+1)\frac{r_o^2}{4a^2}\right] = \max S_r$ when $r_o < 0.588a$	At $q, r < r_o$ $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[4a^2 - (8r^2+4r_o^2)\log\frac{a}{r_o} - \frac{2r^2r_o^2}{a^2} + \frac{r^4}{r_o^2} - 3r_o^2\right]$ At $q, r > r_o$ $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[4a^2 - (8r^2+4r_o^2)\log\frac{a}{r_o} - \frac{2r^2r_o^2}{a^2} - 4r^2 + 2r_o^2\right]$ At center Max $y = -\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[4a^2 - 4r_o^2\log\frac{a}{r_o} - 3r_o^2\right]$ for r_o very small (concentrated load) Max $y = -\frac{3W(m^2-1)a^2}{4\pi Em^2t^3}$
		Uniform on concentric circular ring of radius r_o ⑧	At $q, r < r_o$ $S_r = S_t = -\frac{3W}{4\pi mt^2}\left[(m+1)\left(2\log\frac{a}{r_o} + \frac{r_o^2}{a^2} - 1\right)\right] = \max. S$ when $r < 0.31a$ At $q, r > r_o$ $S_r = -\frac{3W}{4\pi mt^2}\left[(m+1)\left(2\log\frac{a}{r} + \frac{r_o^2}{a^2}\right) + (m-1)\frac{r_o^2}{r^2} - 2m\right]$ $S_t = -\frac{3W}{4\pi mt^2}\left[(m+1)\left(2\log\frac{a}{r} + \frac{r_o^2}{a^2}\right) - (m-1)\frac{r_o^2}{r^2} - 2\right]$ At edge $S_r = \frac{3W}{2\pi t^2}\left(1-\frac{r_o^2}{a^2}\right) = \max. S$ when $r_o > 0.31a$ $S_t = \frac{3W}{2\pi mt^2}\left(1-\frac{r_o^2}{a^2}\right)$	At $q, r < r_o$ $y = -\frac{3W(m^2-1)}{2\pi Em^2t^3}\left[\frac{1}{2}\left(1+\frac{r_o^2}{a^2}\right)(a^2-r^2) - (r^2+r_o^2)\log\frac{a}{r_o} + (r^2-r_o^2)\right]$ At $q, r > r_o$ $y = -\frac{3W(m^2-1)}{2\pi Em^2t^3}\left[\frac{1}{2}\left(1+\frac{r_o^2}{a^2}\right)(a^2-r^2) - (r^2+r_o^2)\log\frac{a}{r}\right]$ At center Max $y = \frac{-3W(m^2-1)}{2\pi Em^2t^3}\left[\frac{1}{2}(a^2-r_o^2) - r_o^2\log\frac{a}{r_o}\right]$

(continued on next page)

Circumstances and Reference Number			STRESS	DEFLECTION
Form of Plate	Manner of Support	Manner of Loading and Reference No.		
		Uniform over small eccentric circular area of radius r_0  (9)	At point of load, f (1) $S_r=-\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a-\rho}{r_0}+(m+1)\frac{r_0^2}{4(a-\rho)^2}\right]$ = max. S when $r_0<0.6(a-\rho)$ At edge (1) $S_r=\frac{3W}{2\pi t^2}\left[1-\frac{r_0^2}{2(a-\rho)^2}\right]$ = max. S when $r_0>0.6(a-\rho)$	At q $y=-\frac{3W(m^2-1)}{2\pi Em^3t^3}\left[\frac{1}{2}\left(\frac{\rho^2r^2}{a^2}-r_1^2\right)-r_1^2\log\frac{\rho r_1}{ar_1}\right]$
		Central couple (trunnion)  (10)	At $r=r_0$ (1) Max. $S_r=\frac{3M}{4\pi t^2r_0}\left[1+\left(\frac{m+1}{m}\right)\log\frac{2(0.45a+r_0)}{0.45ka}\right]$ where $k=\frac{0.1a^2}{(r_0+0.14a)^2}$	
	Supported by uniform pressure over entire lower surface	Uniform over concentric circular area of radius r_0  (11)	At q, $r<r_0$ $S_r=-\frac{3W}{2\pi mt^2}\left\{(m+1)\log\frac{a}{r_0}+\frac{1}{4}\left(1-\frac{r_0^2}{a_2}\right)\left[(m-1)-\frac{(3m+1)r^2}{r_0^2}\right]\right\}$ $S_t=-\frac{3W}{2\pi mt^2}\left\{(m+1)\log\frac{a}{r_0}+\frac{1}{4}\left(1-\frac{r_0^2}{a^2}\right)\left[(m-1)-\frac{(m+3)r^2}{r_0^2}\right]\right\}$ At q, $r>r_0$ $S_r=-\frac{3W}{2\pi mt^2}\left\{(m+1)\log\frac{a}{r}+\frac{1}{4}(m-1)\left[1-\frac{r_0^2}{a^2}+\frac{r_0^2}{r^2}\right]-m+\frac{(3m+1)r_0^2}{4a^2}\right\}$ $S_t=-\frac{3W}{2\pi mt^2}\left\{(m+1)\log\frac{a}{r}-\frac{1}{4}(m-1)\left[\frac{r_0^2}{a^2}+\frac{r_0^2}{r^2}\right]+\frac{1}{4}(m-5)+\frac{(m+3)r_0^2}{4a^2}\right\}$ At center Max. $S_r=S_t=-\frac{3W}{2\pi mt^2}\left[(m+1)\log\frac{a}{r_0}+\frac{1}{4}(m-1)\left(1-\frac{r_0^2}{a^2}\right)\right]$	At q, $r<r_0$ $y=\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[(8r^2+4r_0^2)\log\frac{a}{r_0}+2r^2\left(\frac{m-1}{m+1}\right)\left(1-\frac{r_0^2}{a^2}\right)+2r_0^2\left(\frac{3m+1}{m+1}\right)+\frac{r_0^4}{a^2}-a^2\left(\frac{7m+3}{m+1}\right)-\frac{(a^2-r_0^2)(r^4-r_0^4)}{a^2r_0^2}\right]$ At q, $r>r_0$ $y=\frac{3W(m^2-1)}{16\pi Em^2t^3}\left\{(8r^2+4r_0^2)\log\frac{a}{r}+2r^2\left[\frac{3m+1}{m+1}-\frac{(m-1)r_0^2}{(m+1)a^2}\right]-\frac{r^4}{a^2}-a^2\left(\frac{7m+3}{m+1}\right)+\frac{2(m-1)r_0^2}{m+1}\right\}$ At center Max. $y=-\frac{3W(m^2-1)}{16\pi Em^2t^3}\left[4r_0^2\log\frac{a}{r_0}+2r_0^2\left(\frac{3m+1}{m+1}\right)+\frac{r_0^4}{a^2}-a^2\left(\frac{7m+3}{m+1}\right)+\frac{(a^2-r_0^2)r_0^4}{a^2r_0^2}\right]$ for r_0 very small (concentrated load) Max. $y=-\frac{3W(m-1)(7m+3)a^2}{16\pi Em^2t^3}$
	No support	Uniform edge moment  (12)	At any point $S_r=S_t=-\frac{6M}{t^2}$	At q $y=-\frac{6(m-1)M(a^2-r^2)}{Emt^3}$ At center Max. $y=-\frac{6(m-1)Ma^2}{Emt^3}$
Circular, with concentric circular hole (Circular flange)	Outer edge supported	Uniform over entire actual surface $W=w\pi(a^2-b^2)$ (13)	At inner edge Max. $S=S_t=-\frac{3w}{4mt^2(a^2-b^2)}\left[a^4(3m+1)+b^4(m-1)-4ma^2b^2-4(m+1)a^2b^2\log\frac{a}{b}\right]$ when b is very small Max. $S=S_t=-\frac{3wa^2(3m+1)}{4mt^2}$	At inner edge Max. $y=-\frac{3w(m^2-1)}{2m^2Et^3}\left[\frac{a^4(5m+1)}{8(m+1)}+\frac{b^4(7m+3)}{8(m+1)}-\frac{a^2b^2(3m+1)}{2(m+1)}+\frac{a^2b^2(3m+1)}{2(m-1)}\log\frac{a}{b}-\frac{2a^2b^4(m+1)}{2(a^2-b^2)(m-1)}\left(\log\frac{a}{b}\right)^2\right]$
	Outer edge supported	Uniform along inner edge  (14)	At inner edge Max. $S=S_t=-\frac{3W}{2\pi mt^2}\left[\frac{2a^2(m+1)}{a^2-b^2}\log\frac{a}{b}+(m-1)\right]$	At inner edge Max. $y=-\frac{3W(m^2-1)}{4\pi Em^2t^3}\left[\frac{(a^2-b^2)(3m+1)}{(m+1)}+\frac{4a^2b^2(m+1)}{(m-1)(a^2-b^2)}(\log\frac{a}{b})^2\right]$
	Supported along concentric circle near outer edge	Uniform along concentric circle near inner edge  (15)	At inner edge Max. $S=S_t=-\frac{3W}{2\pi mt^2}\left[\frac{2a^2(m+1)}{a^2-b^2}\log\frac{c}{a}+(m-1)\frac{c^2-a^2}{a^2-b^2}\right]$	
	Inner edge supported	Uniform over entire actual surface $W=w\pi(a^2-b^2)$  (16)	At inner edge Max. $S=S_t=\frac{3w}{4mt^2(a^2-b^2)}\left[4a^4(m+1)\log\frac{a}{b}+4a^2b^2+b^4(m-1)-a^4(m+3)\right]$	At outer edge Max. $y=\frac{3w(m-1)}{16Em^2t^3}\left[a^4(7m+3)+b^4(5m+1)-a^2b^2(12m+4)-\frac{4a^2b^2(3m+1)(m+1)}{(m-1)}\log\frac{a}{b}+\frac{16a^4b^2(m+1)^2}{(a^2-b^2)(m-1)}(\log\frac{a}{b})^2\right]$
	Outer edge fixed and supported	Uniform over entire actual surface $W=w\pi(a^2-b^2)$  (17)	At outer edge Max. $S_r=\frac{3w}{4t^2}\left[a^2-2b^2+\frac{b^4(m-1)-4b^4(m+1)\log\frac{a}{b}+a^2b^2(m+1)}{a^2(m-1)+b^2(m+1)}\right]$ At inner edge Max. $S_t=-\frac{3w(m^2-1)}{4mt^2}\left[\frac{a^4-b^4-\frac{1}{2}a^2b^2\log\frac{a}{b}}{a^2(m-1)+b^2(m+1)}\right]$ = Max. S	At inner edge Max. $y=-\frac{3w(m^2-1)}{16m^2Et^3}\left\{a^4+5b^4+2a^2b^2+8b^4\log\frac{a}{b}+N\right\}$ $N=\frac{\left[8b^6(m+1)-4a^2b^4(m+3)+4a^4b^2(m+1)\right]\log\frac{a}{b}-16a^2b^4(m+1)(\log\frac{a}{b})^2-4a^2b^4+2a^4b^2(m+1)-2b^6(m-1)}{a^2(m-1)+b^2(m+1)}$

(1) Empirical formula based on strain measurements. See "Stresses Produced in a Circular Plate by Eccentric Loading and by a Transverse Couple" R.J.Roark, Bulletin 74, Eng. Exp. Station, Univ. of Wis., 1932
(2) Approximate formula, after Morley. See "Strength of Materials", A.Morley, Longmans, Green & Co., London 1919, p. 441
(3) Approximate formula obtained by interpolation, linear with respect to α, between values for an infinitely long strip and a circular plate
(4) Based on coefficients given by Timoshenko. See "Applied Elasticity", S.Timoshenko and J.M. Lessells, Westinghouse Technical Night School Press, Pittsburgh, 1925
(5) Approximate formula, after Timoshenko. See "Über die Biegung der allseitig unterstützten rechteckigen Platte unter Wirkung einer Einzellast" S.Timoshenko, Der Bauingenieur, Jan. 31, 1922, p. 51
(6) Approximate formula, after Nadai. See "Über die Spannungsverteilung in einer durch eine Einzelkraft belasteten rechteckigen Platte" A. Nadai, Der Bauingenieur, Jan. 15, 1921, p. 11
(7) Approximate formula, obtained by linear interpolation, with respect to α, between values for an infinitely long strip and a square plate
(8) Approximate formula, after Westergaard. See "Moments and Stresses in Slabs", H.M. Westergaard and A. Slater, Proc. American Concrete Inst., Vol. 17, 1921
(9) Approximate formula, after Nadai. See Der Bauingenieur, Vol. V, 1924, p. 1

(continued on page 283)

B.E.C. Machine for Testing the Mechanical Stability of Greases

This article which has been prepared by the Bearing Engineers' Committee of the Anti-Friction Bearing Industries Association, describes a testing machine and procedure to forecast the operating stability of greases to be used for anti-friction bearings

METHODS of studying the probable behavior of lubricating greases, on machines in storage, were described in the June issue of *Product Engineering*. The objective of the tests described in that article was the elimination of the evils resulting from oxidation and deposit formations.

Another active phase of grease research has to do with mechanical stability while in service. Those who use anti-friction bearings in their products have had many reports of trouble which, when investigated, were shown to arise from not using the proper kind of grease. Some grades have a tendency to dry out and harden in service, causing serious damage to the bearing. In some instances the oil and soap separate. The lubricant then runs out and leaves the residue caked in the races. Other types increase in volume and work out of the housings, making their use objectionable when cleanliness is desired.

Procedures for overcoming these mechanical servicing troubles have been receiving considerable attention from the Anti-Friction Bearing Industries Association. All were aware of the existing need for correct specifications. Through their Bearing Engineers' Committee they have appointed a Sub-Committee on Lubrication to study the embarrassing features and to make constructive recommendations to eliminate them.

A complete pooling of data and experience showed that a determination of grease properties other than those customarily reported would be necessary to properly evaluate ball-bearing greases. The bearing manufacturer often does not know the type of machine in which the bearing is assembled, nor the kind of service to which it is put. Both are major factors in specifying the grease to be used. For best results, users of anti-friction bearings should recommend a suitable grease to the purchaser of the machine after making tests of greases, or requiring the grease manufacturer to make them. Only in this manner can they learn what is and is not satisfactory to use with their particular product.

A preliminary survey revealed that many of the bearing companies had individually developed mechanical tests. After two years of exploration work, and study devoted to the merits of existing test equipment, the Bearing Engineers' Committee designed and perfected a composite assembly known as the B.E.C. Grease Test Machine. It is offered to the grease makers and the users of anti-friction bearings, as suitable apparatus to determine the mechanical stability of greases, to supplement the usual chemical tests.

While the lubricating value of the grease is not indicated by this machine, test observations will show comparative starting and running torque, oil separation, change in structure, change in consistency, channeling, leakage past the bearing seal, aeration and the resulting

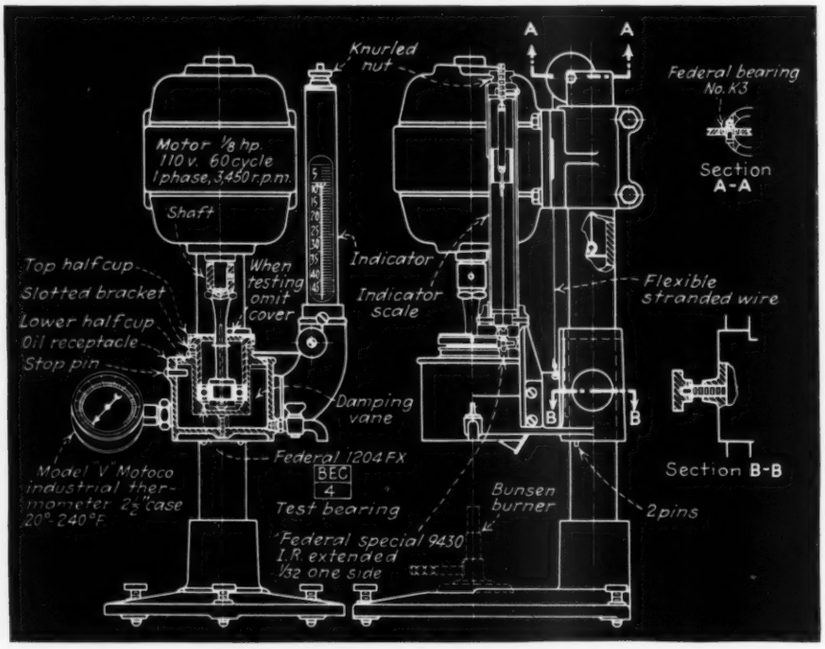


Fig. 2—The grease testing machine for determining mechanical stability as developed by Bearing Engineers' Committee

GREASE REPORT

BRAND NAME .....

MANUFACTURED BY .....

CHEMICAL ANALYSIS:

Soap (nature and per cent)	Water
Oil viscosity at 100°F	Dirt
Free Acid (as Oleic) %	Filler (nature and per cent)
Free Alkali (as NaOH) %	

PHYSICAL PROPERTIES:

Color	Oxidation, Gumming
Odor	Water absorption

Unworked Consistency—BEC

Penetrometer	Bleeding
Melting Point °F	Max. Recommended Temp.
Structure Fibrous	
Structure Smooth	Min. Recommended Temp.
Structure Etc.	
Corrosion	

B.E.C. Testing Machine

	80°–85°F.	150°F.	220°F.
Torque starting (gm.cm.)			
Torque final (gm.cm.)			
Separation (yes or no)			
Structure			

Worked Consistency—BEC

Penetrometer	
Leakage (yes or no)	
Aeration	

Fig. 3—Standard form for recording test data as an aid in comparing successive samples

increase in the volume of the grease.

The machine shown in Figs. 1 and 2 is a simple vertical design, consisting of a base, a vertical column which supports a bracket carrying a constant speed 3,450 r.p.m. motor, and an extension for the motor shaft to which is fastened the inner race of the test bearing. The outer race of the bearing is held integral with the grease cup, which is free to turn. To measure the torque, this cup is connected to the torque scale spring by a flexible cord. The oil bath is adjustable for height and is counterweighted.

The principle of this machine is the working of a definite amount of grease in a moderate size bearing running at a medium speed. The test procedure and operating technique are simple. In setting up for the test the oil receptacle is filled to a depth indicated by a line inside with oil of approximately SAE No. 50. It is important that oil of the same grade and viscosity be used at all times to obtain comparative results. The test bearing, which is a Conrad type, BEC 4, single shielded, SAE 204, and cup are removed from the machine and then washed in a suitable solvent and wiped clean. When reassembled, with the shield at the bottom, the bearing is fastened to the shaft and 1¼ cu.in. of the grease to be tested is packed into the bearing and around the spindle. The grease is leveled off after the cup is put on.

The oil receptacle is raised to the stop on the column with damping vane on test cup in front. The alignment of test cup is adjusted for free rotation and to avoid any interference.

With the cord detached from the test cup the indicator is set at zero by means of the overhead knurled nut provided with a pin which locks in one of six holes. The cord is next attached to the adjustable slotted bracket on the cup so that the indicator shows an initial tension of one-half division on the scale. The indicator scale is graduated in units of 20 grams. The pitch radius of the cord around the cup is 3 cm., so that each graduation on the scale represents a torque of 60 gram-centimeters.

A gas burner is used to heat the oil in the receptacle, to which an indicating dial thermometer is attached. Tests, each continuous for 20 min., are made on calcium soap greases at oil bath temperatures of 80-85, and at 150 deg. F., and on

sodium soap greases at 80-85, 150 and 220 deg. F. Between successive tests the temperature is raised at the rate of 10 to 12 deg. F. per min. The torque is recorded every minute up to 10 and then at 15 and 20 minutes. Observations of separation, change in structure, leakage and aeration are noted while the test is in progress.

A standard test record form is shown in Fig. 3 and indicates desirable data that should be tabulated as a means of making comparisons of various greases. The chemical analysis data usually furnished by the grease manufacturer is used to control the manufacturing process. To the bearing user it is an aid in checking the difference between lots of the same grade and brand of grease.

Color, odor and structure of a sample are indices that point to the presence of tallow, rosin, rosin oil and oils that give off an odor when warm. Very pale greases usually contain low viscosity mineral oils, while dark and black greases generally contain heavy dark lubricating oils.

For determining the melting point the use of the Modified Ubbelohde Melting Point Apparatus is recommended.

To compare the body or density of the grease before and after working, a B.E.C. penetrometer has been developed. This instrument is said to be a more sensitive unit than the A.S.T.M. machine, and is somewhat smaller, as is required by the smaller volume of grease tested. It consists of a suitable grease receptacle, a means for removing the grease sample, a cone-pointed plunger, a dial means for recording the distance the plunger penetrates the grease, a convenient means to adjust the cone point level, a means for tripping the plunger, a stand on which the several parts are mounted in proper alignment, and a swivel-jointed mounting of the dial and plunger to allow them to swing to one side for the convenient filling and cleaning of the grease container.

The mechanical instability of greases which are unsuited for ball-bearing use can be demonstrated with these tests.

Multi-Speed Free-Wheeling Drive for Centrifugals

TO UNLOAD heavy viscous solids or hard-packed crystalline materials from centrifugals requires a low-speed, high-torque drive. But during the extraction period speed is high, torque is low, acceleration must be rapid and starting is frequent. To incorporate these requirements in a two-speed motor was found impractical. To solve this problem the engineers of the American Tool & Machine Company devised a two-motor, overrun drive.

As shown in the accompanying illustration the constant-speed, high-torque motor for unloading, is mounted horizontally, while the rapid accelerating drive motor is in a vertical position. The slow-speed motor transmits its drive to the vertical shaft of the high-speed motor. A free-wheeling or over-running clutch in the gear of the worm drive between the two motors automatically disengages the unloading motor when the higher-speed, main driving motor runs.

The basket speed during unloading is kept constant whether torque requirements are low as when the basket is rotating free, or whether the torque is high, when the unloading plow is taking a full, deep cut.

Stalling or overspeeding is eliminated. The operator can give his entire attention to the action of the discharger and can therefore unload the basket in much less time than if he were attempting to maintain a balance between stalling and overspeeding regardless of variations in cake density. Unloading time has been reduced from $2\frac{1}{2}$ to $3\frac{1}{2}$ min. to less than 30 seconds.

Because each of the motors is designed independently and specifically for the duty it is to perform, the characteristics of each are designed for maximum suitability for the requirements. The frame size of the main motor is smaller than in the case of a two-speed motor with low-speed windings. The main motor is designed to give whatever acceleration, high-speed and frequent start characteristics may be required.

Two motors with over-running clutch connections save time in unloading centrifugals as made by the American Tool & Machine Company

COLOR Takes to the Rails

HENRY DREYFUSS

A complete train is built to satisfy the needs of a particular consumer market

SOON the New York Central "Mercury" train will be streaking over the landscape between Cleveland and Detroit. This entire train has been designed and built to meet the requirements of a particular market and from the beginning the possible customer was kept in mind. The Detroit-Cleveland public had to be sold an idea as well as a service. Every detail has been considered to make travel comfortable, fast and pleasant.

The New York Central had the engineers, the shops and the equipment. It was the task of the industrial designer to work with them in the train's construction and to furnish practical artistic designs and decorative schemes.

There are seven units in the train: engine-and-tender, combination baggage car and coach, combination coach and kitchen, dining car, lounge car with a bar, and the observation car. Each car is sound and shock proofed and air-conditioned. Baggage racks and small crowded vestibules were largely done away with. Many new decorative materials, never before put on trains, were used.

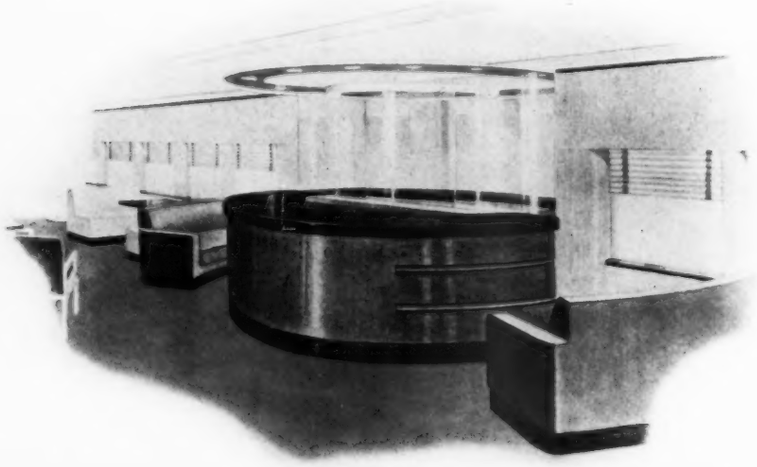
Perhaps the most important task the designer had to face was the color problem. Color makes or breaks first impressions. More than anything else in product design, it creates the atmosphere. The right colors, rightly combined, assured success in providing the best restfulness, hospitality, luxury, and cleanliness in railroad travel. Many color schemes were tried in sketches before final decisions were made. Careful consideration was given every factor affecting the color, lighting, arrangements, and the particular atmosphere desired in each car.

Variety Prevents Monotony

In the Mercury the use of the same general color scheme in all the cars ties up the entire train as a unit. Monotonous repetition is avoided by the great variety of tones and blending shades.

The exterior of the powerful locomotive and smooth streamlined train is in medium gray. The locomotives' driving wheels and all locomotives' trim are in silver, and at night are illuminated by hidden lights to capitalize on the trains advertising value.

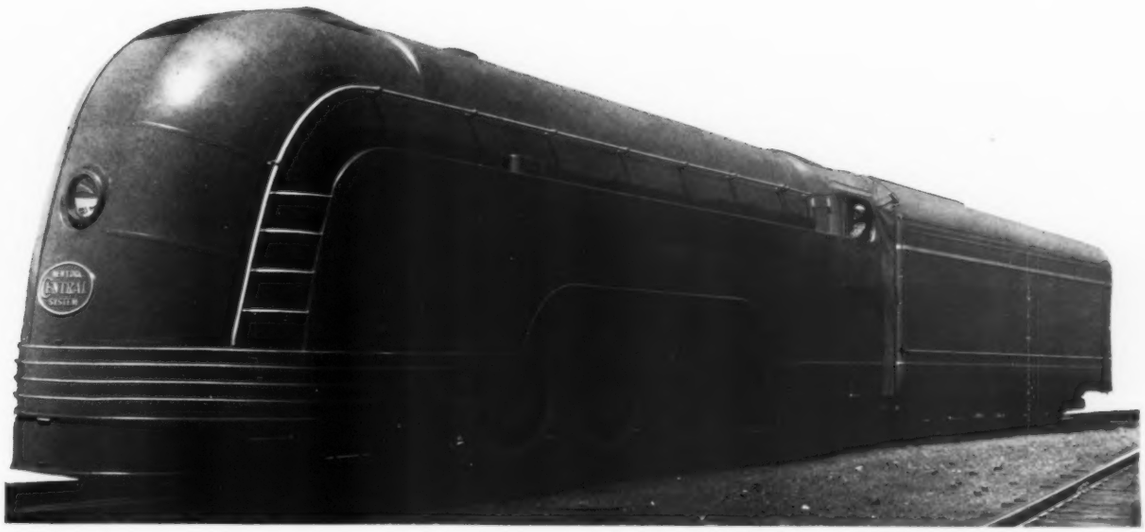
Color psychology, to make travelers feel at home or relaxing in a club, played an important rôle in decisions. Interior color schemes never before used on trains were adopted. Where a gay cheerful atmosphere was wanted, bright and light colors were used. Where restful comfort was the keynote, soft harmonious tones, warm and rich, were selected.



The textures of different materials for tone, shading, and contrast as well as harmony, were combined to give variety of effect. Many of the fabrics were specially dyed to get just the right shade. Special paint colors were developed by the research department of the Sherwin-Williams Company.

In the cars, color heightens the radical changes made in plan and arrangement, gives width and spaciousness and gets away from the regimentation of standard railroad car plans.

The semi-circular vestibules, entirely new in plan, with extra wide



passageways between cars and brilliantly lighted, present an extreme contrast with those of yesterday. There are brilliant red doors, soft gray painted dado and aluminum upper walls and ceiling. Dark blue linoleum is used on the floor with a gray circular inlay in which N.Y.C. is lettered in red.

The luxury, hospitality and conviviality of the lounge car dictated the deep rich shades of brown, gold, tan, rust and blue-green in its color scheme. In the coach, where trim tailoring was in order, clear, clean tones of brown and green were chosen. The long aisle in the coach is broken in the center by a group of four easy armchairs, facing into the car and with convenient tables between them. The specially developed seats are upholstered in a brown diagonal material. The floor is mottled blue-green rubber tile, and blue-green shades are at the windows. The ceiling is light tan enamel. Individual reading lights, individually controlled, are incorporated in the solid aluminum baggage carriers. General illumination is located in a broad band running the entire length of the ceiling.

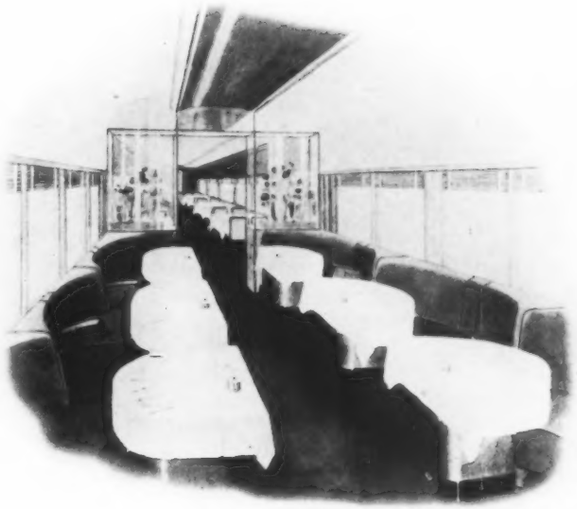
Divided into three sections by large double glass partitions, the dining car is the most unusual feature of the train. The two end sections are grill rooms. The center is the diner in blue and tan, a gay café with a banquette seating arrangement, never before done in a train. A waiting lounge with two large couches is at one end.

In the smoking rooms of the coach cars the coloring is both utilitarian and decorative. The scheme is green, blue and tan with metallic blue painted walls. The leather couches are tan and the chairs blue.

The hospitable lounge car has for its focal point a brilliantly illuminated, mirror-backed, semi-circular bar, located midway in the car. Venetian blinds add a smart luxurious touch.

The semi-circular parlor compartment with a soft-tone cloud photomural on the outside walls is the center of the car. This makes two spacious rooms and puts the compartment in the most comfortable part of the car. Tan, brown and blue-green, with walls of walnut Flexwood, is the main coloring of the compartment. The couches are restful and luxurious.

The parlor and observation cars are done in much the same color scheme as the lounge, but with different



materials and combined in various ways. Large movable easy chairs are upholstered in a great variety of materials, and replace the old-fashioned Pullman seats. Three couches built as a central piece of furniture and placed in the center of the observation end, allow passengers to look out the window instead of in the car.

The men's and women's saloons with new conveniences, good lighting, and new arrangement, are a departure from the dull and dreary rooms found on most trains. The men's saloons are in tones of bright rust, with mottled black and rust-colored, rubber tile floors. Original Rockwell Kent lithographs are on the walls as a decoration. Various shades of peach and rust make the women's saloons bright and feminine, and reproductions of Gaugin watercolors are on the walls.

Every comfort, luxury and convenience that is usually found in a conservative club has been amply provided for on the "Mercury." Those who travel on this streamlined train will appreciate the great advance made in appointments, in furnishings, in floor plan and clear space. All these resulted from a study of the traveller's requirements and then designing and building a custom-made product to suit.

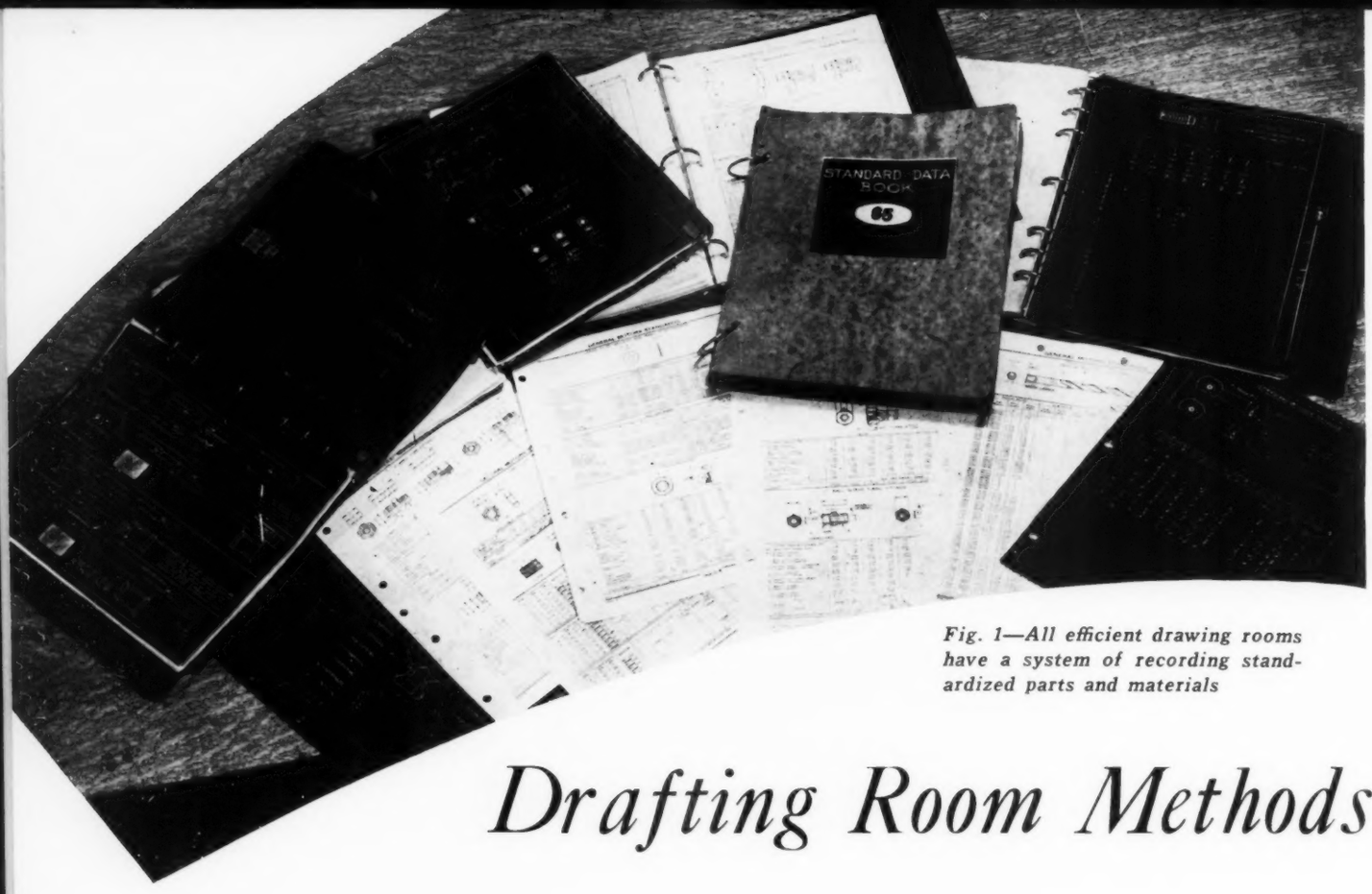


Fig. 1—All efficient drawing rooms have a system of recording standardized parts and materials

Drafting Room Methods

SYSTEM, with its many ramifications, is the backbone of every modern drafting room. Although system is often considered synonymously with "red tape" it is an actual necessity to the orderly functioning and coordination between the drawing room and the shop. Without system there is no common understanding among all of the various departments of a plant.

Although many of the larger concerns operate on successful systems developed during many years of experience, there are also other plants that have no conception of the importance of a systematized plan upon which a drafting department is or should be conducted. The work is usually done in an inefficient and crude manner that in most instances will be an expensive overhead when the concern grows large enough to realize the need of a proper system for this important part of the organization.

To establish such a successful system, care must be taken not to over-systematize. Usually the best systems are the simple ones. However, the system should be broad enough to take care of future needs. When selection is made for the adoption of a given plan to be followed, it is well to consider the cooperation of all other departments that have to work with drawings, as to a large extent the success of the concern may hinge on the choice.

Drawing Forms

Each plant must determine what form and sizes of drawings best suit its products. As to a basic size, letter-size paper, 8½x11 in., as recommended by the American Standards Association, should be used whenever possible. Larger sizes of sheets should be multiples of the 8½x11 in. size.

There are a number of items and notations that may be desirable to incorporate in the drawing form, such as a space for company name, detail designation and drawing and piece number. The drawing number should be shown in two places, at the top and bottom of the right-hand side of the sheet. This is convenient both for filing and for reference. And, as sometimes happens, when a drawing is torn at the corner and one of the numbers is lost, the other will then suffice for drawing identification.

That the draftsman will not overlook some essential information, it is advisable to provide space for the scale size, date, whether made or purchased, finish, heat-treatment, and raw materials with size, kind, temper and weight per 1,000 pieces, as shown in Fig. 2*a-b*. Spaces should, of course, be provided for "drawn by," "checked by," and "approved by." A column designating where the part is to be used is also necessary.

All reference to drawing scale should be uniform. A suggested method is to show 1 = 1 in the box for full scale, 1 = 2 for half size and 2 = 1 for double size. This method eliminates lettering the words. Some drawing rooms prefer 1/1, 1/2, or 2/1 as the scale designation.

Caution notes, as in Fig. 2*c*, as "read all notes before starting work," "do not scale drawing, work to dimensions," and "all dimensions given in inches," will sometimes avoid misunderstandings with the shop and paternmaker. Such notes may seem superfluous, but past experience has shown it best to incorporate them. If a change column is used, a notation that "changes are opposite change letters in margin" can be shown.

There is always opporunity for a difference of opinion regarding dimensional variation that otherwise have no limits. For small and medium size products, such caution notes, as shown in Fig. 2*d*, may read "unless otherwise specified with limits, decimal dimensions may vary +0.005 —0.005 in.;" and another, "fractional dimensions to machined surfaces may vary +0.0 0

-0.010 in." To avoid having such notes misinterpreted or to apply to any other items, a notation "drill, punch, and commercial stock sizes are not included in the general limits" should be shown. This type of information on the drawing form will, of course, vary according to products and production in different plants. Such notes can be incorporated without making the form too elaborate, as the drawing should be the main item on the sheet.

Numbering of Drawings

It is surprising to observe the numerous and different systems that are used for numbering drawings. Many of these systems show considerable originality and initiative. The system of prefixes used in many plants is, in most instances, for the convenience of the man in charge of the drawing file and is used for the purpose of immediately associating a part with some particular product or subject. To prefix parts, such as *H* for head, *V* for vacuum, *P* for pump, or other standard

and Management–II

J. K. OLSEN
Formerly Chief Draftsman, Stewart-Warner Corporation

Standardization of drawings for uniformity in dimensioning and the recording of data with suggested drawing forms, parts lists and revision record sheets

parts, such as screws, nuts and washers, seems to be superfluous, for as a concern grows, and when parts are to be used interchangeably, this numbering system will eventually become a handicap.

As an example of the above, a particular plant had a drawing number with prefix on the drawing indicating the size of sheet, a different number with prefix being used for the production number, the pattern still carrying another number, and the catalog referred to the part with an altogether different number. In this instance, two cross-indexes were needed to find a given piece. To say the least, it was somewhat awkward when everyone used a different number for specifying this particular piece. Obviously, such a condition should not exist. The administrator's duty is to carefully select a system that will serve, regardless how long the concern stays in business. It can be done only by placing this responsibility in the drafting room.

Probably the simplest and most efficient method of numbering drawings, parts, and products is the use of a consecutive numbering system. The drawing of a part should have a number not used for anything else. The pattern should also have the same number, thereby eliminating errors. It should be remembered that such numbers are copied many times on orders and in correspondence, and it will be found that such a consecutive numbering system will not only eliminate cross indexing but will also facilitate handling throughout the plant.

However, there may be some advantage in using a separate and distinctive type of number for production drawing in a large plant where many different types of drawings are made, in that drawings being sorted for filing are recognized at a glance. When assigning numbers for part assemblies, they should be in the same series as the part numbers. Part assemblies may have a prefix, for example, *G*, indicating this to be a group

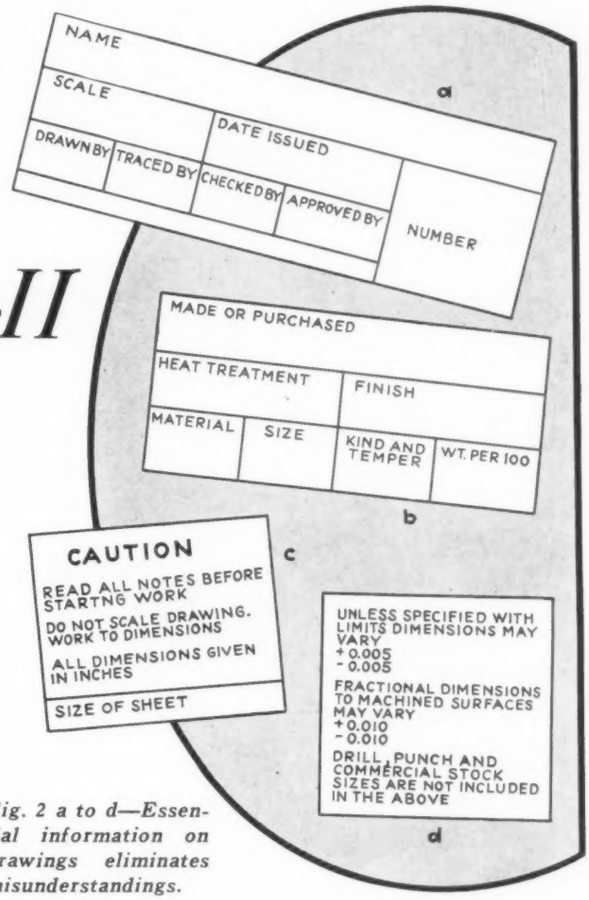


Fig. 2 a to d—Essential information on drawings eliminates misunderstandings.

drawing. Numbers for models are generally assigned with a separate series of numbers usually depending on the product that is made.

Uniform drawings, through standardization, should be the aim of every drafting room. Confusion of standards and methods has been eliminated through the progress made by the American Standards Association in the issuance of a booklet, "Drawings and Drafting Room Practice." It is the purpose of this association to so systematize the general appearance of a drawing, regarding arrangement of views, line work, dimensioning and lettering, that drawings made in one plant are easily translated in another plant. Such standardization eliminates the necessity of breaking in a new draftsman. It will also help the beginner by learning but one system which is recognized throughout the country.

In many drafting rooms it is the practice to include inch marks with dimensions. This seems to be a waste

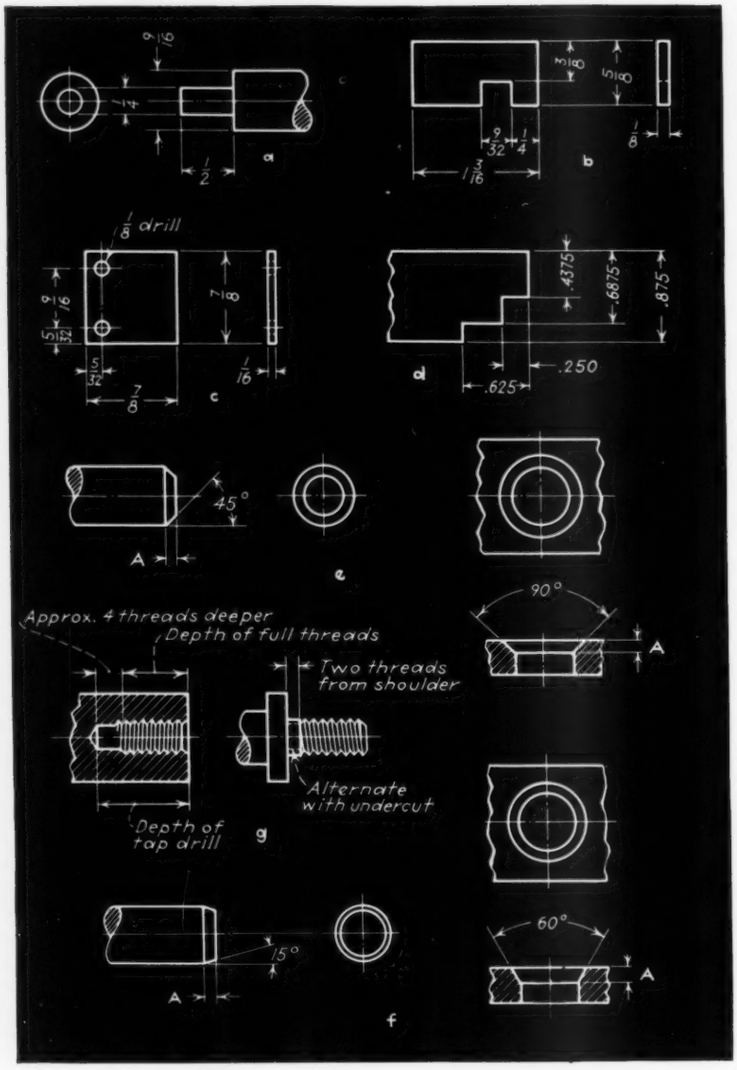


Fig. 3 a to g—Suggested standards of dimensioning and the application of notes on shop drawings

of time, not only for the draftsman but for the checker as well. On drawings where all dimensions are in inches, inch marks can be eliminated by inclusion of a note printed on the drawing form, "All dimensions given in inches." However, in drawings where long dimensions are involved, both feet and inches should be specified. The foregoing is in accordance with the American Standards Association recommendations.

For simplification of dimensioning, fractional sizes should be specified whenever possible, as shown in Fig. 3*a-b-c*. Unimportant dimensions, such as inside and outside diameters, flat and overall dimensions, fillets and radii, rod and hole sizes, and location of holes from locating surfaces can be given in fractions. In the design of drawn or formed parts, or for flat stampings or punchings, the use of common fractions facilitate the making of tools and avoids delay and expense of special tools. In many places $\frac{1}{64}$ sizes are eliminated entirely. In general manufacturing, limits are usually plus and minus 0.010 in. for fractions.

Many times it is necessary to hold fractional dimensions to closer limits than a customary plus and minus 0.010 in., and in this instance these fractional dimensions are given in decimals, as shown in Fig. 3*d*. Such dimensions show the shop and inspection department that the tolerances should be held within a narrower margin, generally plus and minus 0.005 in. or any other specified range.

Decimal Dimensions

When decimal figures of fractions are given, the "rounded-off" system should be used. For example, $\frac{3}{32}$ of which the decimal is 0.09375 should be rounded-off to 0.094. It is obvious that when limits of plus and minus 0.005 is applied to such a dimension, it is better to assume 0.089 to 0.099 than 0.08875 to 0.09875. The general rule for rounded-off figures is to use the next greater figure when the decimal is more than 5 at the fourth figure to the right of the decimal point.

There is, of course, an exception to the above rule, especially in pitch diameters of gears, pitch and lead of screw threads, or for dimensions where gages will be used. In these instances, dimensions can be given to 4, 5, or 6 decimal places, not that an inspector is required to maintain the given dimensions, but because it is the correct figure, no other figure sufficing.

When rod ends or holes are to be chamfered to remove the burr, the dimensions and angle should be shown as in Fig. 3*e-f*. While the size of the chamfer is governed by requirements, dimension *A* may range in the following sizes of 0.005, 0.010, $\frac{1}{64}$, $\frac{1}{32}$, $\frac{3}{64}$, $\frac{1}{16}$, $\frac{3}{32}$, and $\frac{1}{8}$ in. In some shops the actual width of chamfer is shown parallel to the angle. The American Standards Association recommends giving the angle and diameter at the large end of the chamfer for holes.

When noting threads and tapped holes, as in Fig. 3*g*, specifications for drill and tap sizes, number of threads per inch, depth of drilled hole and threaded portion should be given. Lettered notes on drawings should be placed closest to the subject to which they refer and not to be longer than 4 in. per line.

Specifications

Not only should general drawing procedure be standardized, but also that pertaining to technical data, such as standards parts, raw materials, heat-treatments and finishes. Standard parts are usually tabulated under their respective headings for the purpose of ready reference. These lists are then used throughout the plant to avoid duplication of parts.

As an example of standardization, where a plant makes its own washers, tabulated forms enable the designer to select punch and die sizes already available for a variation of washer thicknesses, thereby saving additional tool costs. The same is true of machine screws. It might be desirable to use American National Standards for screws, but to confine the choice to but one series, either coarse or fine. This also applies to various classes of fits. The use of such stand-

ardized screws not only keeps stocks within a limited range, but larger lots of screws could be made at one time and a better stock of fewer taps kept available. Such a standard must originate in and be maintained by the drafting room.

Notes for heat-treatment should conform to the nationally recognized standards, although many companies have special heat-treatment for special materials. These, however, should be limited.

Raw Materials

Because of the various gages of different metals, such as strip steel, sheet brass, wire, plate, and other forms, such materials should be tabulated to show suitable ranges of preferred thicknesses that are available. This allows the designer to use parts that are made of approximately the same size of stock.

In one plant a preferred series of steel strip was used with thickness dimensions of 0.020, 0.025, 0.032, 0.042, 0.049, and 0.065 in. For brass the thicknesses were 0.020, 0.025, 0.0319, 0.040, 0.0508, and 0.064 in. Both materials were similar in thickness so that a change could be made from one material to the other without a change in design.

Many plants keep the information on raw material separate from the drawings. This information is usually kept on cards and filed in drawers. In other plants, it is entered in books and classified according to a chosen system. There may be some good reason for so handling this information, but the fact remains that the raw material specification is an engineering function and with the improvements and increase in number of kinds and grades of modern material it is now a complicated and specialized problem even for the engineering staff. Substitutions of or other questions on raw materials can be controlled by the engineering department where this function rightfully belongs.

Record Sheets

Although in some plants assembly drawings have a space allotted for the names of the pieces pertaining to the particular assembly, many plants also use separate forms for parts, part assemblies, and models. This part list or list of materials, usually lettered on vellum, can be blueprinted and sent to the various departments for cost estimating, ordering-in, and purchasing. Such ruled sheets have spaces for the name of the part, material from which the part is made, piece number, and number required for the machine or unit.

The same procedure is true of recording changes in designs and dimensions. Many drafting rooms require that such alterations be noted in a space provided on each drawing. It is important to select a simple method of recording changes on drawings so that it will be easy to maintain the record without too much "bookkeeping." One of the favored systems is to note the changes by consecutive letters, starting with the letter *A*, in a column incorporated on the drawing form, as shown in Fig. 4*a*. The change of the detail itself is noted with some letter within a circle adjacent to the altered dimension or shape.

To keep the face of the drawing free from change reference letters, another method is to place the letter in one or both longitudinal and vertical borders of the drawing, as in Fig. 4*c*. The intersection of these letters locates the change in a manner similar to that used in locating the names of cities on a map. Because the edges of drawings get the most wear, especially through filing, it might be well to place these reference letters about ½ in. inside the border lines. This method, while eliminating the cluttering-up of dimensions with change references, also facilitates locating the altered portion of the drawing more readily.

Still in other plants, the changes are recorded in a space devoted to such notes, without the use of reference letters, shown in Fig. 4*b*. It is felt that in this way the changes can be described in more detail.

A system of changes that should be avoided is one of issuing a changed drawing but with a new date. Such drawings having the same drawing number are filed together and it is too easy to get hold of the wrong drawing. When a part is so changed or redesigned that it is not interchangeable with a former part, a new piece number should be assigned. When parts are no longer used, drawings should be conspicuously marked with the words "obsolete" or "cancelled," as shown in Fig. 4*d-e*. The information should contain the number of the piece by which it is superseded.

Other plants record changes on separate 8½x11-in. forms on paper that can be blueprinted. These sheets of "change authorization" are numbered and dated and have space provided for the name of the piece and description of the change. The changes are divided into five general classes as follows:

1. Change at once. No shipment to be made without this change.
2. Change at once, but not to affect finished product.
3. Change at once, but not to affect finished product or parts which will delay important orders.
4. Change at once, not to affect parts made or those in stock. Stock to be balanced and all parts to be used up as per estimate.
5. Change to be made at the convenience of the production and manufacturing departments.

The class shown at 4 above is probably the most common change that is used.

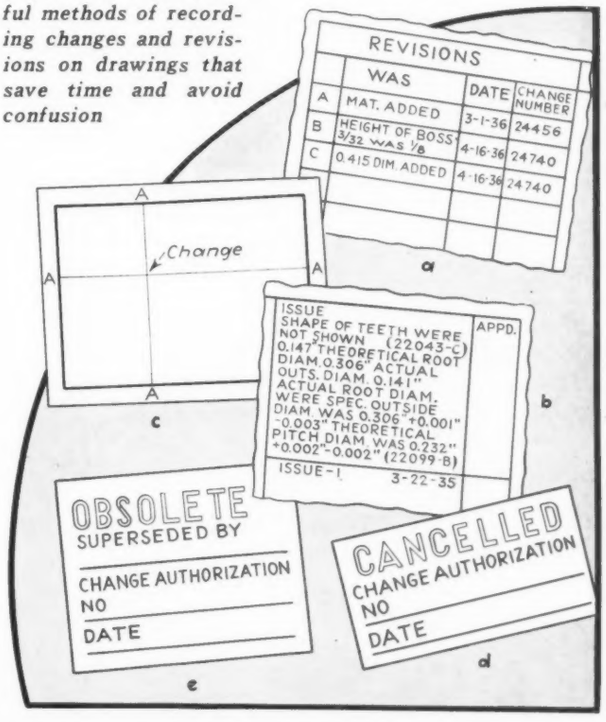


Fig. 4 a to e—Successful methods of recording changes and revisions on drawings that save time and avoid confusion

Analysis of Materials By the X-Ray Diffraction Method—II

HERBERT R. ISENBURGER
St. John X-Ray Service, Inc.

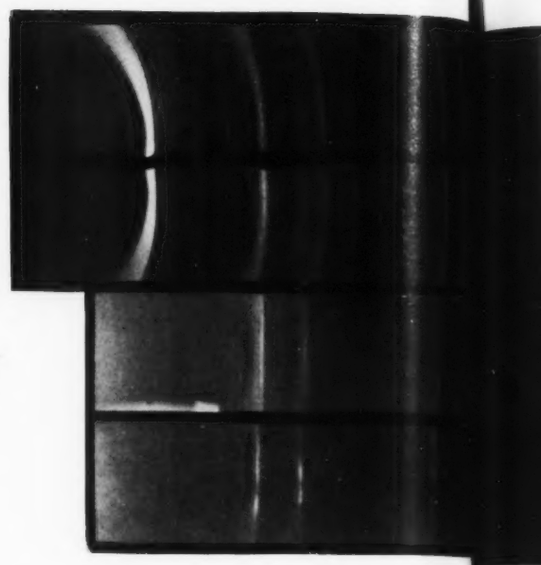


Fig. 1a—Corresponding lines of equal intensity indicate identical structures. (b) Two alloys of slightly different composition

How X-ray diffraction analysis discloses composition, grain size, atomic structure, cold working, heat-treatment and fatigue properties of materials

INTERPRETATION of diffraction patterns involves a study of the position of the lines, the sharpness or diffuseness of the lines, and the relative intensities of the different lines in a single pattern. In comparing different materials the significant effects are the variations in the relative intensities of corresponding lines in different patterns and the variations of intensity of different parts of the same line. As already pointed out the angle of diffraction can be determined from the position of a line which is measured as the distance from the zero line to the reference point on the line. The atom spacing can then be calculated according to Bragg's diffraction equation.

Any change in the position of a line means a change in the corresponding spacing in the particles of the material. No change can take place in one family of planes without similar changes in all other families, except those normal to the first family. For two atomic arrangements to be completely identical it is thus necessary for all lines in the pattern to coincide exactly with the corresponding lines in the pattern of the other, as in Fig. 1*a*. Lack of correspondence, such as shown in Fig. 1*b* indicates some difference in structure.

If proportional changes take place for all families the ratios for the various spacings are not changed, the patterns remain similar, but all the lines are shifted, as in the case of two alloys of different composition illustrated in Fig. 2*a*. The same is true if two elements have the same crystalline form. Thus α-iron and tungsten both crystallize in the body-centered cubic system, and as shown in Fig. 2b, the two patterns are similar. However, the tungsten atoms are larger and the planes further apart so that all the angles in the tungsten pattern are similar and the lines are nearer the zero line.

Although there are more than 30 types of crystal structure, of the materials more commonly used in engineering practice only three kinds are of immediate importance. They are the (1) face-centered cubic crystals, (2) body-centered cubic crystals, and (3) hexagonal crystals. To the first group belong metals which are ductile, such as copper, aluminum, silver, gold, lead, nickel and austenitic iron. To the second group belong α-iron and the brittle metals, such as molybdenum, tungsten and chromium. In the third group are zinc, osmium, and a few others.

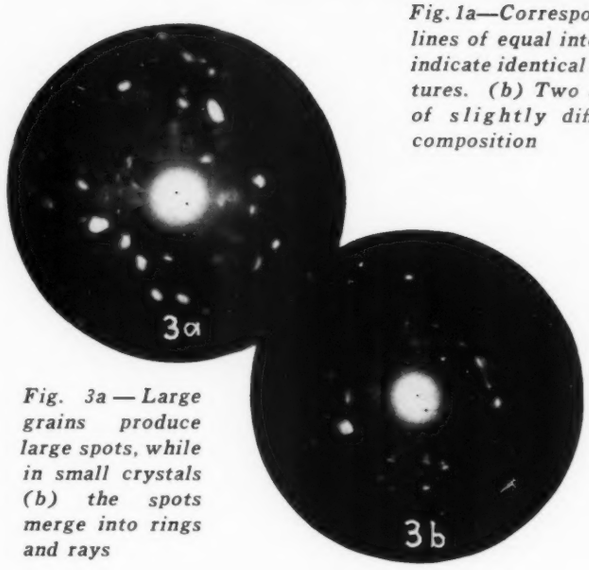


Fig. 3a — Large grains produce large spots, while in small crystals (b) the spots merge into rings and rays

The character of the lines, whether continuous and diffuse, continuous and sharply defined, or consisting of single spots, depends upon the size of the particles causing the diffraction.

A single crystal in a fine beam gives rise to a more or less symmetrical pattern of spots. Hence, the portion of the sheet brass represented by the pattern of Fig. 3*a* is a single crystal with an axis nearly parallel to the X-ray beam. Several crystals together give rise to overlapping sets of patterns as illustrated in Fig. 3*b*. A better idea of the relation between particle size and the appearance of the patterns can be secured from Fig. 4, which shows a series in which the particles range from those visible under a high power microscope

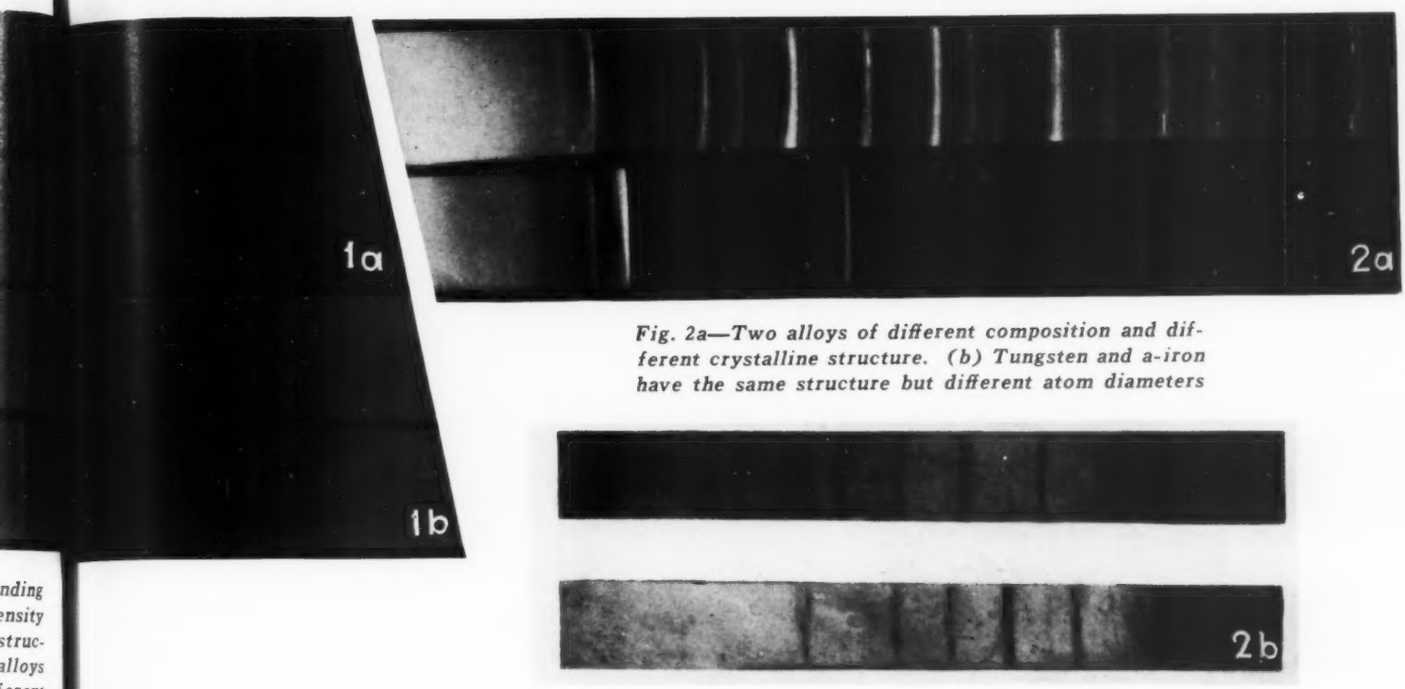


Fig. 2a—Two alloys of different composition and different crystalline structure. (b) Tungsten and a-iron have the same structure but different atom diameters

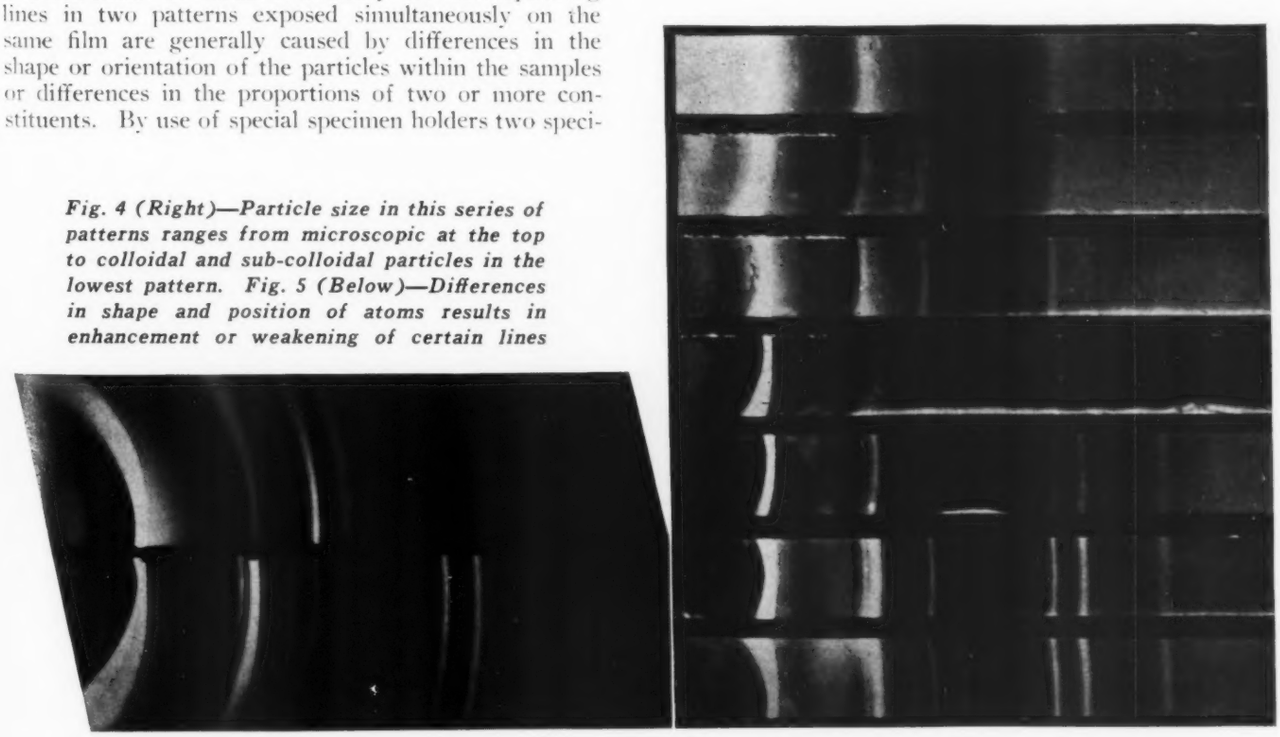
through particles of colloidal dimensions to sub-colloidal particles which are those only a few atoms in diameter, shown in the patterns at the bottom.

The relative intensities of the different lines in the same pattern depend primarily upon the atomic arrangement within the grains. In the simplest possible arrangement, (the simple cubic) atoms are placed at the common corners of an array of adjacent cubes, forming three families of refracting planes. If single atoms are replaced by groups of atoms certain lines are enhanced or weakened, giving opportunity for phase differences between unequally spaced planes in a family.

Differences in relative intensity of corresponding lines in two patterns exposed simultaneously on the same film are generally caused by differences in the shape or orientation of the particles within the samples or differences in the proportions of two or more constituents. By use of special specimen holders two specimens can be mounted and exposed at the same time for exact comparisons, and the two possible causes of differences between the two samples are readily distinguished. Difference in shape or position of the particles produces varying degrees of enhancement or weakening of the various lines as shown in Fig. 5. Comparative enhancement or weakening of groups of lines would indicate differences in the composition of the particles.

Distortion of the particles within the material is also detected readily by the pinhole method. The distorted particles cause radial streaks or "asterism" in the pat-

Fig. 4 (Right)—Particle size in this series of patterns ranges from microscopic at the top to colloidal and sub-colloidal particles in the lowest pattern. Fig. 5 (Below)—Differences in shape and position of atoms results in enhancement or weakening of certain lines



tern as a result of straining the particles beyond their elastic limit. The effect upon the pattern of soft sheet brass, Fig. 6*a*, as a result of flexing it several times is shown in Fig. 6*b*. This method lends itself to the examination of entire sheets of formed articles which can be mounted in the instrument without destroying them and is especially suited and applicable to engineering materials.

In Fig. 7 the patterns illustrate the structural changes taking place in high-speed steel when subjected to progressive heat-treatment. A study of this pattern shows that the quenching operation produces some austenite as well as ferrite and that this austenite persists until the material is again heated or "drawn" above 500 deg. C. (930 deg. F.). This heat-treatment also tends to increase the grain size of the ferrite as shown by the increased sharpness of the lines.

Another method of analysis, the back-reflection method, recently has been developed. This is a modification of a method introduced some years ago by Sachs, that makes it possible to detect the causes leading to fatigue failure. The interpretation of the resulting X-ray pattern is not an easy task and much more work is required before this method can be applied to routine examination.

With the present development of X-ray diffraction analysis the designer now can know many more facts about the materials he uses. The radiologist does not know all the problems of product design, but he can offer his rays as a new tool to the designer.

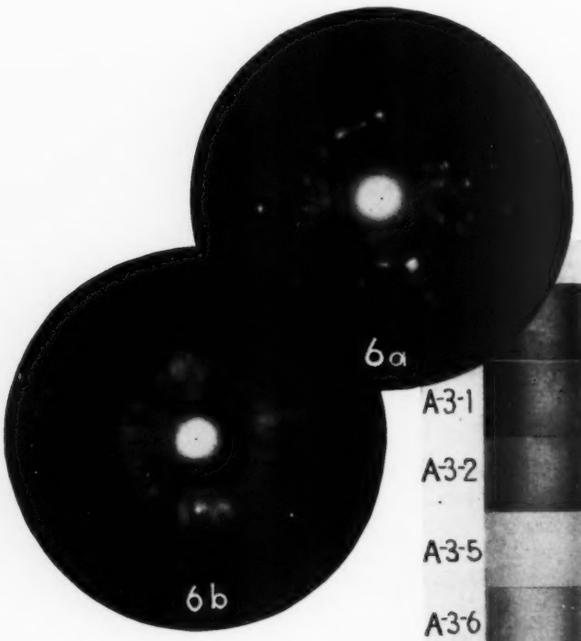


Fig. 6 — Cold working of the brass specimen at (a) resulted in the asterism or streamers shown in (b) for same material

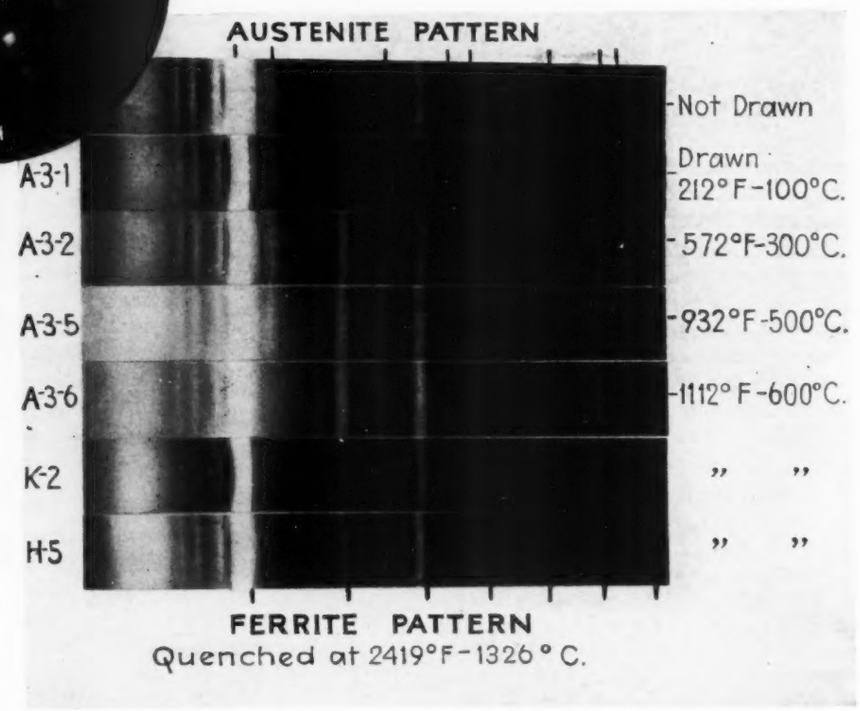


Fig. 7—Effect of heat-treatment of steel is to weaken the austenite lines and strengthen the ferrule lines

Trends in Gear Materials

THERE is a decided trend toward the addition of molybdenum to alloy steels for gears. SAE 4640 is used for Hupp Motor Car Company transmission gears. New Britain Gridley Machine Company is favoring moly-nickel gears, U.S. Electrical Manufacturing Company uses chrome-moly for the driving pinion of its Syncrogear, White trucks and buses use nickel-chrome-moly. It was reported that a set of nickel-chrome-moly automobile transmission gears after having run 100,000 miles were still in serviceable condition.

Marine reverse and reduction gears by Snow & Petrelli Company are of SAE 2345 and 3140. Gears of SAE 3140 and 3312 are used by Wright Aeronautical Corporation. SAE 2512 is another popular steel for aircraft and truck engines.

Single helical spur gears enjoy a continuing wider use, superseding the straight type. Among others, a power shovel manufacturer has recently adopted helical gears. Along with this there seems to be a decided trend toward anti-friction bearing mountings. In the case of single helical gears, rarely does the designer use them without anti-friction bearings.

The main advantages of the single helical gears, according to R. S. Marthens, of the Nuttall Works of the Westinghouse Electric & Manufacturing Company include uniform tooth loading, uniform wear and high impact resistance.

How to Make Alignment Charts

M. G. VAN VOORHIS

Further applications of parallel or perpendicular solution lines and various methods of determining the curve scale for nomograms having more than one function of any variable

PART V

IN NOMOGRAMS involving either parallel or perpendicular solution lines, it is obvious that the relation of similar triangles applies and that differences in the size of the scales can be expressed mathematically by the proper evaluation of constants introduced into the equations. A nomogram that is similar to that shown in Fig. 29 on page 186 of the May number, can be made of the following equation except for the angular relationships.

Nomogram for $f_1(x)/f_2(y) = f_3(u)/f_4(v)$

In Fig. 30, the two lines, intersecting at any angle, are subdivided with uniformly divided scales of $f_1(x)$ and $f_2(y)$ so that zero of each scale is at the intersection as at a in the illustration. The size of each scale is independent of the other and may be as large as the space allows. Either the opposite sides of the same two lines, or two other lines intersecting at the same angle and either parallel to the original lines as at b or perpendicular as at c and d are subdivided with uniformly divided scales of $f_3(u)$ and $f_4(v)$ in the same relative positions so that zero of each scale is at the intersection. These scales need not be the same size as the $f_1(x)$ and $f_2(y)$ scales, but must have the same relative size to each other so that $m_x/m_y = m_u/m_v$ where m is the size of the scale units.

The foregoing procedure is illustrated in Fig. 31 relating volume and temperature of gases, the pressure being constant, as in the equation $v_1/v_2 = (273 + t_1)/(273 + t_2)$ where t = centigrade temperature. If a larger angle were drawn between scales, in this instance a much smaller scale of t would be necessary. The nomogram is shown complete with the centigrade temperature scales t_1 and t_2 replacing the function scales of absolute temperature, the limits of which are 373 to 483, $f(t) = 273 + t$ or 273 + 100 to 273 + 210, as illustrated.

Combination nomograms consisting of the above type and other forms with parallel or perpendicular solution lines are also possible. The nomogram for determining bolt diameters in Fig. 27 on page 186 of the May number is a good example.

The reciprocal scale is to be avoided, although there are some instances when the limits are such that it is not objectionable. The following method avoids three reciprocal scales at once. It is well to recognize that another form of the equation $z = xy/(x - y)$ can be reduced to the type form by first solving for the x value.

Nomogram for $\dfrac{1}{f_1(x)} + \dfrac{1}{f_2(y)} = \dfrac{1}{f_3(z)}$

or $f_3(z) = \dfrac{f_1(x)\ f_2(y)}{f_1(x) + f_2(y)}$

When laying out a nomogram for such equations two intersecting lines are drawn as shown in Fig. 32. Each of the uniform scales of $f_1(x)$ and $f_2(y)$ is subdivided independently of each other and of any convenient size such that zero of each scale will be at the intersection.

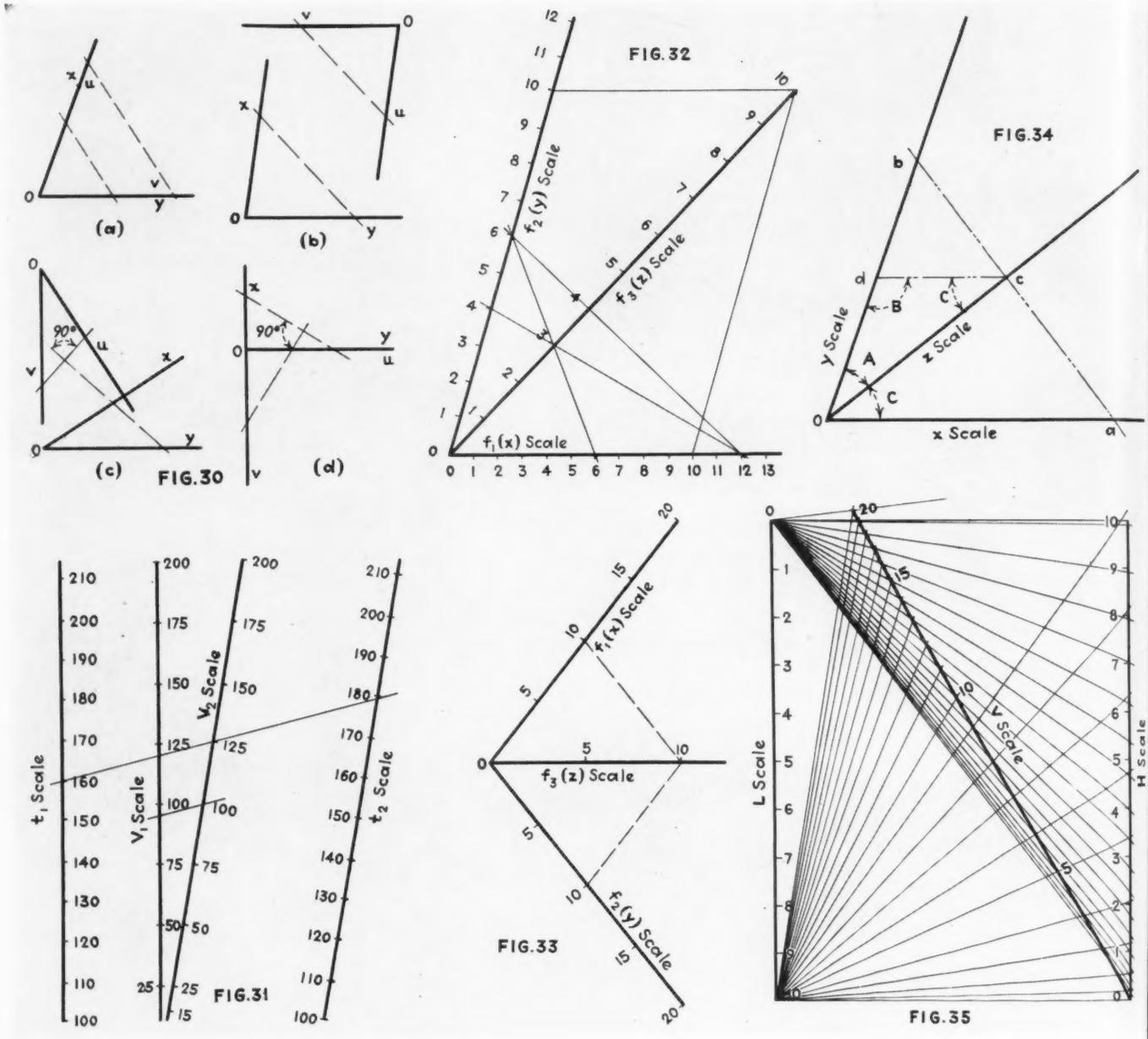
To locate the z scale line, select a value of $f_3(z)$ beyond the range of the $f_3(z)$ value, such as *10* in the nomogram, and draw lines from the same values of $f_1(x)$ and $f_2(y)$, the former parallel to the $f_2(y)$ scale line and the latter parallel to the $f_1(x)$ scale line. The intersection of the two lines locates the selected value of $f_3(z)$ on the $f_3(z)$ scale. The $f_3(z)$ scale line is a diagonal of the parallelogram so formed.

To obtain the size of the $f_3(z)$ scale unit, the $f_3(z)$ scale is subdivided uniformly between the selected value and zero at the intersection of the three scale lines. It is evident from Fig. 32 that no value of $f_3(z)$ beyond 6 can be used.

This type of nomogram is suitable for determining the value of electrical resistances in a parallel circuit, focal length of lenses, and similar formulas. Fig. 33 is another form of Fig. 32 with an obtuse angle between the $f_1(x)$ and $f_2(y)$ scales.

The relationship of angular solution lines to given scales is shown in Fig. 34. The following relations are derived from similar triangles as illustrated by the equations $Oa/Ob = dc/db = dc/(Ob - Od)$. Applying the law of sines to the triangle Odc, then $dc/\sin A = Oc/\sin B = Od/\sin C$. From the last equations, expressions for dc and Od can be obtained in terms of Oc, namely, $dc = (\sin A/\sin B)Oc = K_1Oc$ and $Od = (\sin C/\sin B)Oc = K_2Oc$, where K = the constant. Letting $x = Oa$; $y = Ob$; and $z = Oc$, the original equation can be written $x/y = K_1z/(y - K_2z)$ or $xy - xK_2z = yK_1z$. Dividing by xyz and rearranging the equation, then $(K_1/x) + K_2/y = 1/z$. It will be seen that any straight line across properly drawn scales of x, y and z will solve this type equation.

The procedure as given above for constructing the nomogram in Fig. 34 includes an arbitrary choice of values for K_1 and K_2 in the determination of the sizes of the $f_1(x)$ and $f_2(y)$ scales. It would be possible, knowing the ratio of K_1 and K_2 to compute the position of the $f_3(z)$ scale line. $K_1/K_2 = dc/Od$ so that $K_1/K_2 = 2/3$ approximately, which is the ratio of dc to Od in the nomogram.



Nomograms with Curved Scales

Thus far in this series, nomograms having only straight-line scales have been shown and the corresponding equations or formulas have contained but one function of any variable. Curved scales are used in the following nomograms for equations having more than one function of any variable.

Nomogram for $f_1(u)\ f_3(v) + f_2(u)\ f_4(w) = 1$

Such equations or formulas are not usually written in the above form, but are commonly written $f_3(v) = f_1(u) \pm f_4(w)\ f_2(u)$, which is reducible to the type form by dividing through by $f_1(u)$ and rearranging the terms. Reduction to the type form is required to determine the proper function scales of $f_3(v)$ and $f_4(w)$. To illustrate this reduction, thus, in the equation $C = 0.2416L/log_{10}\ (2L/d)$, the functions C and d are not the usable function, but $1/C$ and $log\ d$ are usable functions. Rewritten the equation becomes:

$$\left(\frac{1}{C}\right)\left(\frac{0.2416L}{\log\ 2L}\right) + \left(\frac{1}{\log 2L}\right)(\log d) = 1$$

The simplest construction of a nomogram of this type is by the graphical method. Two possibilities are available with parallel scale lines for v and w scales, namely, uniform scales of $f_3(v)$ and $f_4(w)$ ascending in the same direction or in opposite directions. The author has found it preferable in most instances to use the arrangement by which the u or curved scale will be placed between the v and w scales, this being determined by trial.

Graphical Method

When using the graphical method, as shown in Fig. 35, two parallel lines are first drawn. One scale for $f_3(v)$ is subdivided uniformly within the limits given and the other similarly for $f_4(w)$ so that the two scales ascend in opposite directions. One point is located on

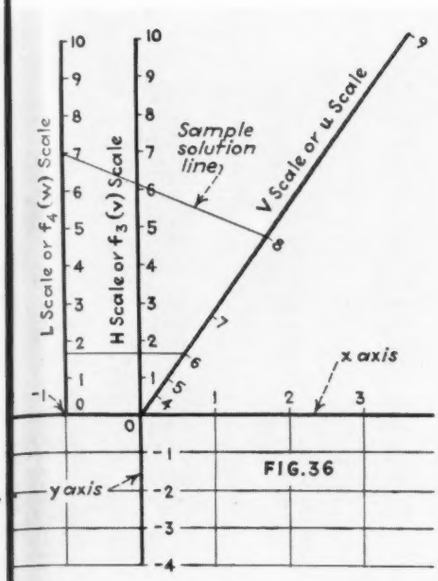
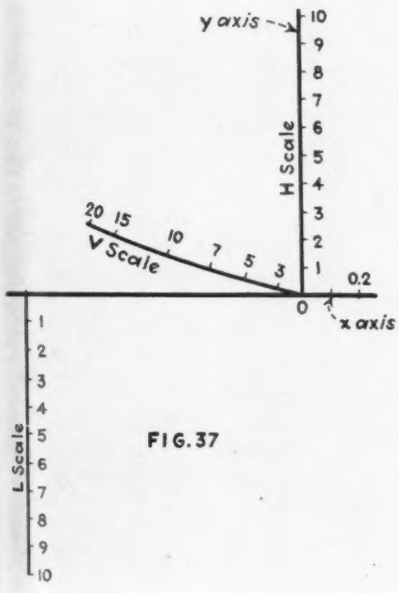


FIG. 30
Parallel solution lines are used at a and b; while at c and d perpendicular solution lines are used

FIG. 31
Nomogram for relation of volume and temperature of gases when the pressure is constant



FIGS. 32 and 33
In a nomogram having either an acute or obtuse angle between the two indicating scales, the z scale is the diagonal of the parallelogram

FIG. 34
Relations of equation Oa/Ob = dc db = dc/(Ob—Od) shown by similar triangles. The law of sines can also be applied

FIG. 35
Graphical method of laying out a nomogram for obtaining the curved scale

FIG. 36
The curved scale is plotted on graph paper from a table of values when the indicating scales ascend in the same direction

FIG. 37
When the indicating scales ascend in opposite directions, the signs are changed and the curved scale is then plotted

the u scale as directed in the following paragraphs, and if this point lies outside the parallel scale lines, one scale is reversed so that the $f_3(v)$ and $f_4(w)$ scales ascend in the same direction. The size of each scale is independent of the other and may be as large as the space allows for the given limits.

To locate the u scale line, a value of $f_3(v)$ and $f_4(w)$ is selected and the original equation solved for $f_4(w)$ *or* $f_3(v)$, respectively, for a set of values of u, which are to be placed on the u scale. The process is repeated for another value of $f_3(v)$ or $f_4(w)$ using the same values of u. The intersections of lines corresponding to the same value of u will locate values of u on the u scale. Function scales may then be replaced with scales of the v and w variables.

To illustrate the foregoing two paragraphs the formula $H = 0.0274\,V^2 + 0.0141\,LV^{1.83}$ gives the friction losses in condenser tubes L feet long when the velocity of water is V feet per second. Written in the type form, the formula becomes

$$\frac{H}{0.0274\,V^2} - \frac{0.0141\,LV^{1.83}}{0.0274\,V^2} = 1$$

L and H can, therefore, be used as they appear in the formula.

In Fig. 35 uniformly divided scales of L and H are drawn on parallel lines with scales ascending in opposite directions. A check computation of H for the value of $V = 10$ when $L = 0$ and when $L = 10$ reveals that the V scale will lie between the L and H scales. A set of values of H can then be determined from the equation, $H = 0.0274\,V^2$ when $L = 0$, and another set from the equation $H = 0.0274\,V^2 + 0.0141 \times 10\,V^{1.83}$ when $L = 10$. A series of values is tabulated below in which H_0 corresponds to $L = 0$ and H_{10} to $L = 10$. In Fig. 35 is shown the manner of plotting the V scale.

V	H_0	H_{10}	V	H_0	H_{10}
1	0.0274	0.168	11	3.315	14.63
2	0.1096	0.528	12	3.945	17.20
3	0.2466	1.162	13	4.63	20.03
4	0.438	2.030	14	5.37	22.94
5	0.685	3.37	15	6.16	26.11
6	0.987	4.73	16	7.02	29.61
7	1.343	6.30	17	7.92	33.02
8	1.754	8.08	18	8.88	36.73
9	2.22	10.06	19	9.89	40.69
10	2.74	12.27	20	10.96	44.76

The following series of examples illustrates the type of equation which can be solved nomographically by this method and also gives the transformation of the equation to the type form.

1. Radius R of a circle having a chord of length c and sagitta, h.

$$R = \frac{c^2 + 4h^2}{8h} \qquad (R)\,(2/h) + (c^2)\left(-\frac{1}{4h^2}\right) = 1$$

2. Stadia formula for horizontal distance in which H is the horizontal distance in feet. R is the rod reading multiplied by *100*, a is the vertical angle, and c the instrument constant.

$$H = R\cos^2 a + c\cos a \qquad H\left(\frac{1}{c\cos a}\right) + R\left(-\frac{\cos a}{c}\right) = 1$$

3. Quadratic equations, both of which will be shown in nomogram form further on in this series.

$$x^2 + Px + Q = O \qquad \frac{P}{-x} + \frac{Q}{-x^2} = 1$$

$$cx^2 + x + d = O \qquad c(-x) + d\left(\frac{-1}{x}\right) = 1$$

4. Kepler's equation (important in astronomy) in which nt and θ are angles in radians and e is the eccentricity of the orbit of a planet.

$$nt = \theta - e\sin\theta \qquad nt\left(\frac{1}{\theta}\right) + e\left(\frac{\sin\theta}{\theta}\right) = 1$$

5. A trigonometric equation

$$a \tan x + b \sec x + 1 = 0 \qquad a(-\tan x) + b(-\sec x) = 1$$

6. A power equation

$$bx = a^x \qquad \log a\left(\frac{x}{\log x}\right) + \log b\left(\frac{-1}{\log x}\right) = 1$$

7. Hydraulics. Average velocity, V in feet per second through a rectangular orifice under low head. The depth from the surface of the liquid to the bottom of the orifice is h_1 and to the top, h_2. Acceleration of gravity, g, is the constant.

$$V = \frac{2\sqrt{2g}}{3}\left[\frac{\sqrt{h_1^3} - \sqrt{h_2^3}}{h_1 - h_2}\right]$$

$$I\left[\frac{h_2}{\frac{2}{3}\sqrt{2g}\ \sqrt[3]{n_2^2}}\right] + \left[N\frac{-1}{\frac{2}{3}\sqrt{2g}\ \sqrt[3]{h^2_2}}\right] = 1$$

The same equation can be written for h_1 supplying the same variable N. The equations are identical and the values of h_1 and h_2 are on the same curved scale. The scale for N may be omitted in the final chart. A nomogram of the accompanying equation will be illustrated further on in this series.

8. Volume of a frustrum of a pyramid where h is the height, A_1 the area of the top and A_2 the area of the bottom. A nomogram of this equation will be illustrated further on in this series.

$$V = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

$$\frac{3V}{h} = \frac{\sqrt{A_1^3} - \sqrt{A_2^3}}{\sqrt{A_1} - \sqrt{A_2}} \qquad \left(\frac{3V}{h}\right)\left(\frac{1}{A}\right) + N\left(\frac{-1}{\sqrt{A^3}}\right) = 1$$

The Plotting Method

In the eight preceding examples it will be noted that any variation in signs in the type form of the equation is applied to functions of u; that is, if the equation, for example, were normally written $f_1(u)\ f_3(v) + f_2(u)\ f_4(w) = -1$, it is changed to the form, $-f_1(u)\ f_3(v) + [-f_2(u)]\ f_4(w) = 1$. The equation then has positive functions of v and w, indicating scales of $f_3(v)$ and $f_4(w)$ ascending in the same direction.

For locating and subdividing the u scale, as in Fig. 36, let x and y represent the following values:

$$x = \frac{-f_2(u)}{f_1(u) + f_2(u)} \text{ and } y = \frac{1}{f_1(u) + f_2(u)}$$

A table is made of the values of x and y for each value of u which is desired to show on the u scale line and the values plotted on graph paper in rectangular coordinates. The size of a unit of x is the distance between the parallel scale lines for the v and w scales and the size of a unit of y is identical with that of a unit of $f_3(v)$ and $f_4(w)$ value.

When locating the v and w scale lines as in Fig. 36, the $f_3(v)$ scale line is uniformly divided on the ordinate or y axis of the coordinates on which the u scale is plotted, with zero at the origin or intersection of the x and y axes. The $f_4(w)$ scale is uniformly divided on a line parallel to the y axis and one unit to the left in the second and third quadrants. The equation for this line is $x = -1$. Zero of the w scale line is on x axis.

The size of the $f_3(v)$ and $f_4(w)$ scales is identical and may be as large as the space allows. A satisfactory distance between the two scales may be determined by the available space and by noting from the table of values of u, x and y, the space taken by the location of the u scale on the coordinates. In this example the portion of the u scale shown is in the first quadrant, but it may lie in any quadrant; in fact, in this instance the u scale passes out to infinity and returns in the third quadrant for values more than nine.

In Fig. 36, the same equation as in Fig. 35 is taken as the example. The formula $H = 0.0274\ V^2 + 0.0141\ LV^{1.83}$, divided through by $0.0274\ V^2$ and rearranged, becomes:

$$\left(\frac{1}{0.0274\ V^2}\right)H + \left(\frac{-0.0141\ V^{1.83}}{0.0274\ V^2}\right)L = 1$$

The value of x will be $0.0141\ V^{1.83}/1$ —$0.0141\ V^{1.83}$ and that of y will be $0.0274\ V^2/1$ —$0.0141\ V^{1.83}$. The $f_3(v)$ scale is the same as the H scale and $f_4(w)$ is the same as the L scale.

The points of the v scale are plotted from data given in the following table, which were computed from the above equations:

V	x	y	V	x	y
1	0.0143	0.0278	6	0.601	1.58
2	0.053	0.1154	7	0.986	2.67
3	0.1181	0.276	8	1.73	4.79
4	0.217	0.534	9	3.67	10.35
5	0.366	0.936			

If the scales of $f(v)$ and $f(w)$ ascend in opposite directions, the type form of equation is rewritten, as $f_1(u)\ f_3(v) + [-f_2(u)]\ [-f_4(w)] = 1$. The equations for x and y are determined as for Fig. 36, so that the example would be:

$$\left(\frac{1}{0.0274V^2}\right)H + \left(\frac{0.0141\ V^{1.83}}{0.0274\ V^2}\right)(-L) = 1$$

With this change in signs, x and y become

$$x = \frac{-0.0141\ V^{1.83}}{1 + 0.0141\ V^{1.83}} \qquad y = \frac{0.0274\ V^2}{1 + 0.0141\ V^{1.83}}$$

The $f_3(v)$ scale is the same as the H scale and $f_4(w)$ is the same as the $-L$ scale.

In Fig. 37 are shown points plotted from data given in the following table which were computed from the above equations.

V	x	y	V	x	y
1	—0.0139	0.027	10	—0.488	1.403
3	—0.0953	0.223	15	—0.667	2.05
5	—0.211	0.540	20	—0.772	2.50
7	—0.332	0.898			

Since these few points determine the scale line closely enough, additional and intermediate values of V may be located by using the H and L scales and the original equation. Thus, when $V = 8$ and $L = 5$: $H = 0.0274 \times 8^2 + 0.0141 \times 5 \times 8^{1.83} = 1.7536 + 3.1684 = 4.92$. Tabulated values of V may also be checked by this method.

Since L is negative in this instance, positive values of L are plotted in the third quadrant. This usually makes a nomogram of inconvenient proportions. Since it is desirable to use a method which will confine the plotted curve between the parallel lines, means will be considered to bring $f(v)$ and $f(w)$ opposite each other.

QUESTION AND COMMENT

Liability of Manufacturer for Delay in Shipping Machine Parts

LESLIE CHILDS
Attorney and Counselor-at-Law

● Under ordinary circumstances, mere delay on the part of a machine manufacturer in shipping replacements for his product may not render him liable for special damages. At most, the manufacturer's liability would be confined to the cost of the parts involved. But circumstances may alter cases.

When a machine is sold under a contract that obligates the manufacturer to replace defective parts, a different situation arises. If the manufacturer is notified of the probable loss to a buyer for unreasonable delay in supplying replacements, he may be held liable for special damages caused by such delay.

Obviously, here is a point of law to be considered by machine manufacturers in general in such a situation. The following case is reviewed to show judicial reasoning.

The plaintiff purchased a refrigerating plant from the defendant. Under the contract of sale, the defendant agreed to replace any parts of the machinery that proved defective within one year from date of sale, providing the defective parts were returned with transportation charges prepaid.

Within the year, certain parts proved defective, and the plaintiff called upon the defendant to replace them, in accordance with the contract. Plaintiff also notified defendant of the condition of his business and that unless the parts were shipped promptly he would suffer serious loss by reason of the shut-down of his plant.

At this time there was due an installment note on the original equipment, and the defendant demanded that this be paid, and further, that the plaintiff pay for the new parts, refusing to ship parts until these conditions were met. Plaintiff paid the amount demanded, namely, $465.60. Following this, defendant delayed forty-six days in shipping the needed replacements.

Following payment, the plaintiff sued for damages, first for the price of the new parts, and secondly for loss caused by the shut-down of his plant because of defendant's delay in shipping parts. Defendant claimed he was entitled to payment for the new parts because no defects were disclosed in the parts returned for which he was responsible. Further, that he was not liable for delay in shipment, because under the contract of sale his liability was expressly restricted to furnishing duplicate parts.

On the facts as outlined, a jury awarded the plaintiff the sum of $1,500. This included both the price of the new parts, and the loss caused plaintiff by the shutting down of his plant. The defendant appealed, and the higher court in reviewing the record affirmed the judgment.

The higher court said in part: "Plaintiff was compelled to have the new parts at once. He notified defendant of the damages he would suffer if his plant continued to be shut down for lack of these parts. Apparently a new contract of sale was made. Plaintiff, on demand of defendant, and as a condition precedent to the shipment of the duplicate parts, had to pay an installment note due on the first purchase and to pay in cash the price of the new parts. When this was done there arose the reciprocal obligation of the defendant to ship the parts in reasonable time.

"The defendant especially complains that the plaintiff, even if entitled to recover for the price of the new parts, was not entitled to recover consequential and speculative damages. The complaint definitely stated the claim of right for recovery of damages and for the breach of the contract caused by the negligent failure of defendant to ship in reasonable time the parts the plaintiff had been compelled to buy and pay for.

"Speculative damages are not recoverable, but in this instance there is evidence of definite and concrete losses of a special nature which plaintiff suffered by reason of his refrigeration plant being shut down. Defendant had had ample warning of the danger of these special damages, if there was undue delay in making the shipment. Such damages were within the scope of the pleadings and were justified by the proof. The judgment is affirmed."

Obviously, since each instance of this kind must necessarily be decided in the light of the particular facts involved, the subject cannot be covered by a hard and fast rule. However, as an example of judicial reasoning in which the facts were held sufficient to support a judgment against the manufacturer for special damages for unreasonable delay in making shipment, the case reviewed is of force and value.

Governor-Controlled Motors

● In connection with the article "Governor-Controlled Fractional-Horsepower Motors" on pages 216 to 218 inclusive of the June number, that type of motor has a number of features in its favor that should be mentioned. They are as follows:

1. Large reserve of maximum output and starting torque at low operating speeds in a minimum amount of motor volume.
2. Excellent speed regulation.
3. Adjustment of operating speeds either while the motor is running or stopped.
4. Low cost where salient-pole motors are feasible and where it is unnecessary to go to a high degree of refinement in the speed-control mechanism and cushion mounting.
5. Universal operation within the limitations as stated in the article.

In contrast with the favorable features are certain limitations which may be summarized as follows:

1. Short life as compared to induction motors.
2. Low efficiency.
3. Radio interference.

In applying governor-controlled series-wound motors, the service problem and the cost must be weighed carefully against the advantages obtainable with this type of motor. The motors are admirably suited to a start-stop application requiring fast acceleration, high momentary load capacity and starting torque, universal characteristics, and adjustable speed when necessary. The added advantage of speed control has been a determining factor in applying this type of motor to other applications which operate more continuously, and such motors, when carefully designed and

applied in small sizes, are giving a good account of themselves.

To date the applications on which the larger volume of governor-controlled series-wound motors are used have required motors which do not have a maximum output in excess of approximately 1/15 hp. and an average load much below this figure.

—R. W. GROOT
General Electric Company

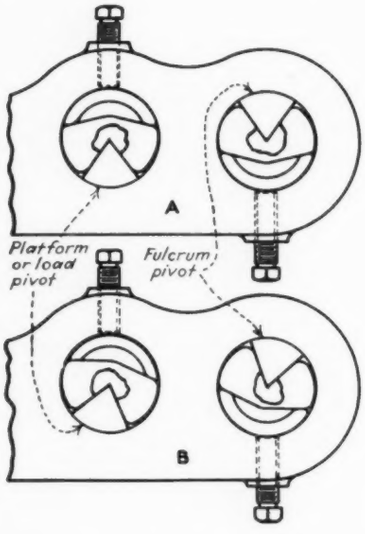
Self-Centering Knife-Edge Pivot

GEORGE GROCOTT
Toledo Scale Company

● To make replacement in the field easier, quicker, and with factory accuracy, the pivot in the Toledo full floating scale platform is self-centering and self-aligning. Originally pivots were cast into the lever and the knife edges filed to a given dimension between the load and fulcrum pivot points. When the knife edges wore, a whole new assembly was necessary. Later on the pivots were driven or forced into the lever with resulting faulty accuracy.

In the new design, the back of the heat-treated tool steel pivot is ground to the same arc as the retainer hole in the lever, the knife edge coinciding with the exact center of the hole. At *A* in the diagram, the pivots are in the normal upright position, the pivot knife-edges coinciding with the vertical center lines. At *B*, the pivots are turned out of normal position, but still the knife-edges coincide with the vertical center lines.

In either position, the horizontal distance of the knife edges is exactly the same. Hence the lever multiple remains constant which is absolutely necessary for scale accuracy. The sharp edge of the pivot under the clamp is rounded off to secure positive clamping action. The retainer block is machined to fit the pivot.



Pivot knife-edge is always at true center of hole regardless of assembled position, thereby maintaining required horizontal center distance

Errata

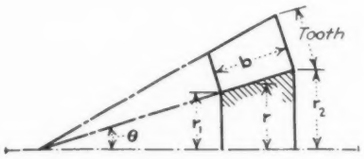
● The heading of the first vertical column in Table I "Thickness of leather for V-Packings" on page 140 of the April issue should be "Width of Packing" instead of "Piston Diameter."

Bevel Gear Teeth

H. R. LLOYD and
FRANK A. MICKLE

● Many authors have published methods of determining the bending strength of bevel gear teeth, some of which are correct and some not. However, in many instances, these authors have used cumbersome and indirect methods of arriving at their final equations. Herewith is a simple direct analysis which to our knowledge has never been published.

It is agreed that when a bevel gear is considered statically, with the top edge of one tooth in contact with the base of the mating tooth, the load distribution is proportional to the radius and the maximum bending stress is constant across the width of the tooth. We are basing our analysis on this basic idea.



Tooth diagram for load intensity at any point along a pitch element

The intensity of pressure at any point along the face of the tooth may be deduced from the torque equation.

w = the intensity of the load on the tooth and pitch cone at radius r
r = the radius of the pitch cone at any point
r_2 = the pitch radius at large end of gear
r_1 = the radius of the pitch cone at small end of gear
b = the width of gear face
T = the total torque on the gear
a = the constant of proportionality
$w = ar$

Then the total torque may be expressed

$$T = \int arr\,db = \int_{l}^{2} a r^2 \frac{db}{dr}\,dr \qquad (1)$$

$$db/dr = b/(r_2 - r_1) \qquad (2)$$

$$T = \int_{l}^{2} a \frac{b}{r_2 - r_1} r^2\,dr \qquad (3)$$

$$T = a \frac{b}{r_2 - r_1} \frac{(r_2^3 - r_1^3)}{3} \qquad (4)$$

$$a = \frac{3T\,(r_2 - r_1)}{b\,(r_2^3 - r_1^3)} \qquad (5)$$

$$w = ar = \frac{3T\,(r_2 - r_1)}{b\,(r_2^3 - r_1^3)}\,r \qquad (6)$$

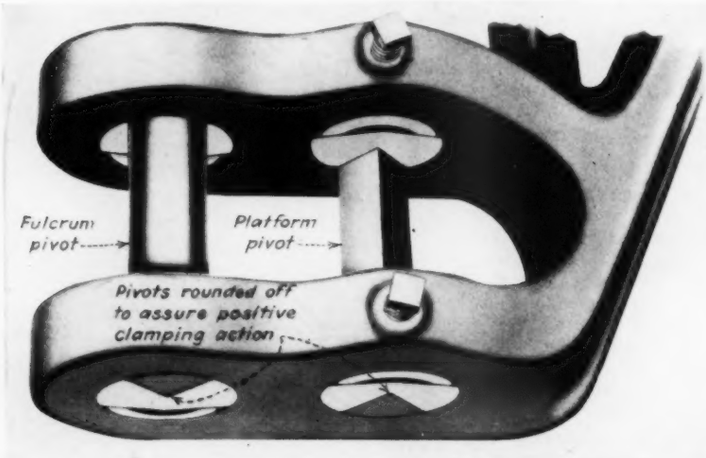
If r is taken as the mean radius $(r_2 + r_1)/2$, Equation (6) reduces to

$$w_m = \frac{3T\,(r_2^2 - r_1^2)}{2b\,(r_2^3 - r_1^3)} \qquad (7)$$

Where w_m is the intensity of pressure at the mean radius or if r if taken as the pitch radius r_2 Equation (6) reduces to

$$w_2 = \frac{3T\,(r_2 - r_1)\,r_2}{b\,(r_2^3 - r_1^3)}$$

where w_2 is the intensity of pressure at r_2.



Equation (6) then permits the use of the intensity of pressure at any point for the design of the tooth providing all dimensions of the cross section of the tooth are measured at that point and the actual width of face b, taken as the width for calculations.

Irregular Plane Sections

ROLAND V. HUTCHINSON
Lansing, Mich.

● Planimetric integration affords a rapid and fairly accurate approach to the subject without involving tedious layout work and leaves a resulting record in a form readily filed for future reference. The procedure is relatively simple.

Given the section the properties of which are sought, draw it to any scale m times size. Cover it with a sheet of Cartesian coordinate tracing paper placing the edges of the grid in a position tangent to the section outline, and trace the section. Note the actual ordinates, mentally square them and point them off on the tracing paper to any new scale n, where n is length of new ordinate per unit value of the squared number. Join the several points to make the closed "square-curves" illustrated in the lower left-hand corner of the illustration, and with a planimeter measure areas of section and "square-curves" as A_1, A_{2x}, A_{2y} respectively.

The area A_s of the actual section is, obviously

$$A_s = A_1/n^2 \qquad (1)$$

Area moments M_s of actual section about its chosen perpendicular tangents are given by:

$$M_{1x_s} = A_{2x}/2m^3n \qquad (2)$$

$$M_{1y_s} = A_{2y}/2m^3n \qquad (3)$$

Coordinates of centroid of the actual section are:

$$x_s = A_{2y}/2m\ n\ A_1 \qquad (4)$$

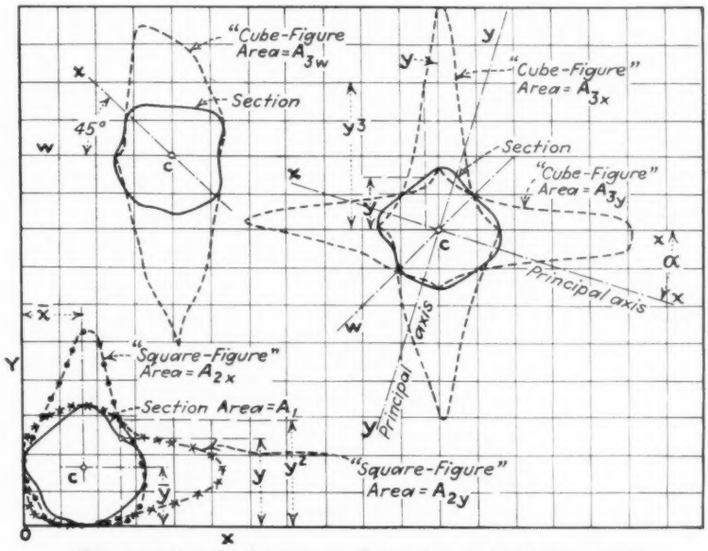
$$y_s = A_{2x}/2m\ n\ A_1 \qquad (5)$$

Coordinates of centroid of section as traced are:

$$x = A_{2y}/2n\ A_1 \qquad (6)$$

$$y = A_{2x}/2n\ A_1 \qquad (7)$$

Having located the centroid, to obtain moment of inertia about an axis passing through the centroid, shift the tracing paper so that a principal gridline of the Cartesian paper passes through the centroid in the required direction. Then cube the ordinates of the section and point off to scale as before and join the points to form the cube-figure and measure its areas as A_{3x} or A_{3y}, as pertains.



The section is drawn on Cartesian coordinate paper tangent to edges of grid. Ordinates are squared and cubed, pointed off and drawn in to any n scale

Then moments of inertia for the actual section are:

$$I_{xc} = A_{3x}/3m^3n \qquad (8)$$

$$I_{yc} = A_{3y}/3m^3n \qquad (9)$$

Integrals evaluated are of type $\int y^n dx$; the method does not directly integrate the form $\int xydA$, the general form of the expression for the product of inertia. When moments about and directions of principal axes are sought, the circumstance that products of inertia vanish for principal axis conditions, enables the use of the following artifice.

After obtaining I_{xc} and I_{yc} for axes mutually perpendicular and intersecting at the centroid, shift the paper again rotating it anti-clockwise through 45 deg. and repeat obtaining the value I_{wc}. The principal axes are now xx, yy, and xx includes with xc the angle α measured clockwise from xc, and

$$\tan 2\alpha = \frac{2I_{wc} - (I_{xc} + I_{yc})}{I_{yc} - I_{xc}} \qquad (10)$$

The principal moments of inertia then become:

$$I_{xx} = \tfrac{1}{2}[I_{xc} + I_{yc} + (I_{xc} - I_{yc}) \sec 2\alpha] \qquad (11)$$

$$I_{yy} = \tfrac{1}{2}[I_{xc} + I_{yc} - (I_{xc} - I_{yc}) \sec 2\alpha] \qquad (12)$$

The least radius of gyration is obtained conventionally from Equations (1) and (11) and (12) herewith.

To illustrate scale effect, suppose the section being studied is drawn 4 times or m size and the square or cube-figures are pointed off so that 100 units of the squared or cubed number are taken as one unit length n, on the corresponding ordinate. Then $m = 4$ and $n = 100$, hence $m^3n = 6400$, this value appearing as a factor in the denominator. The other factor, 2 or 3 respectively is merely the integration coefficient.

Pressed Steel Possibilities

To the Editor:

● The trend toward the use of plate and sheet steel construction in place of castings, recalls an attempt some 15 or 20 years ago in Monroe, Mich., toward such design. A gasoline motor was largely made of pressed steel, the theory being to secure strength and lightness, as well as economy.

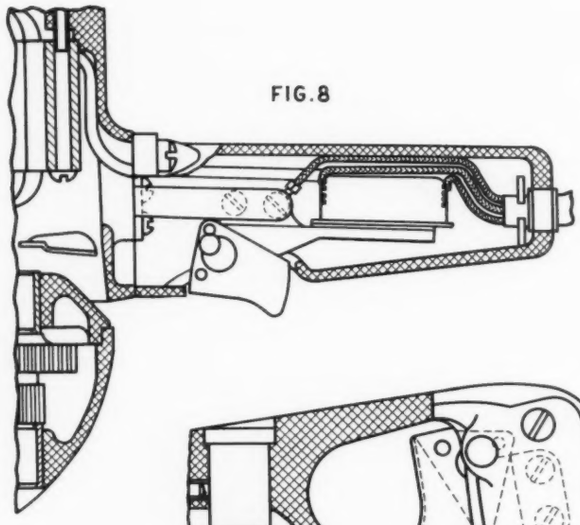
It never reached production volume where economy could be realized by making dies that would secure the lowest cost output. The connecting rods were ingenious pieces but probably could be improved upon with our present knowledge of both stamping and welding.

There might be possibilites for more research and experiments with pressed steel in modern aviation engines, parts of which require excessive machining. The forged steel master connecting rod now used in a certain engine weighs 64 lb. in the rough and less than 15 lb. when finished. Some other parts lose an even greater percentage of weight in machining. Is it not possible in view of new developments, that engineers and designers might study the possibilities of pressed steel to advantage for many types of machinery?

—FRANK C. HUDSON
New York, N. Y.

Switch Handles

B. B. RAMEY
Chief Engineer
Black & Decker Manufacturing Company



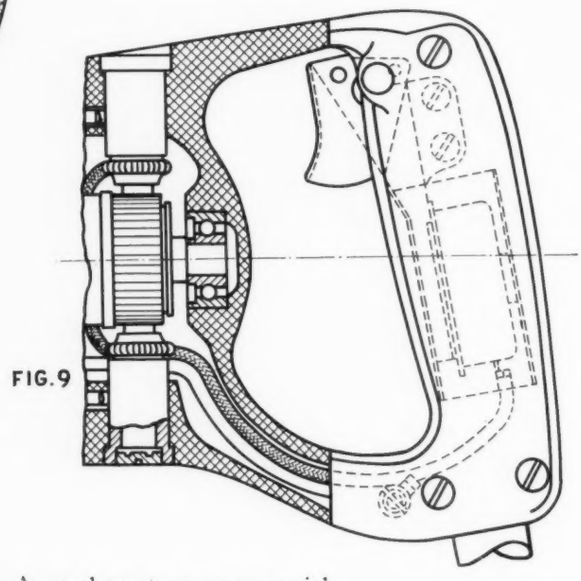
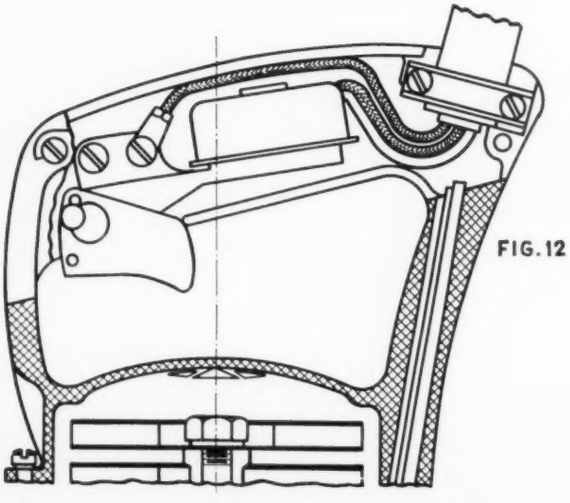


Fig. 8—A modern type commercial switch mounted in a side handle. Such switches are readily replaced as a unit, are inexpensive and sealed against the entrance of dirt. The switch opens as soon as the trigger is released unless the locking pin is set, in which case a slight pull on the trigger releases the locking pin and opens the switch.

Fig. 9—Another example of a modern commercial switch mounted as a unit in a grip type end handle.



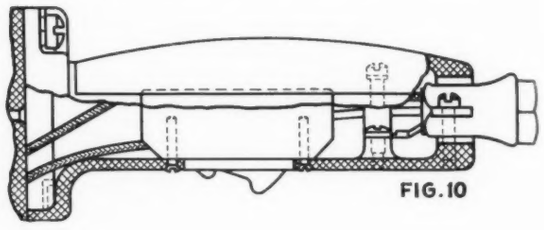


Fig. 10—A rocker type switch such as used on polishers and portable sanders. It is not self releasing and is now being replaced by plunger-operated dust-tight switches such as shown in Fig. 14.

Fig. 11—Another style of mounting a commercial type switch in a side handle. The switch is replaceable as a unit and is self-opening, as soon as the trigger is released; the return spring being shown dotted.

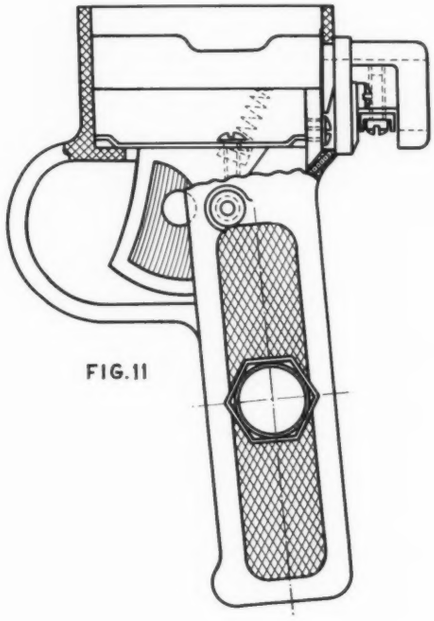


Fig. 12—In this switch mounting the trigger actuates the switch by means of a lift rod attached to the back of the trigger. A tension spring attached to the upper end of the lift rod and anchored to the lower end of the switch plate pulls the switch open as soon as the trigger is released. If the locking pin is depressed when the trigger is pulled back it passes through the hole in the trigger which then cannot return to the open position. As soon as the trigger is pressed the locking pin is released, snaps back and releases the trigger.

and Switch Mountings—II

Fig. 13—A slider operated switch. The slider moves back and forth as indicated in the drawing. This switch is not provided with any release arrangement. It is used only on light model tools where no damage would be done if the tool were laid down with the power still on.

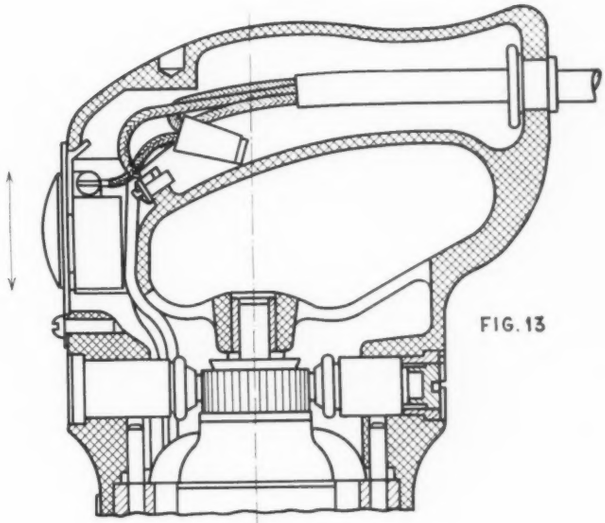


FIG. 13

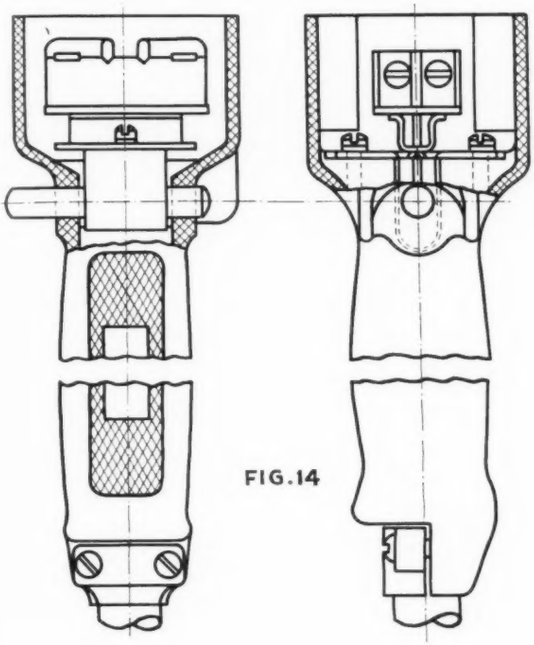


FIG. 14

Fig. 14—Latest type switch handle for polishers, sanders, and portable grinders. The switch is in a dust-tight chamber and is operated by a plunger instead of a trigger, which eliminates the necessity of an opening such as is required when triggers or rockers are used. The plunger makes a close fit. Switch is not self-releasing, it being necessary to push the plunger for both on and off positions.

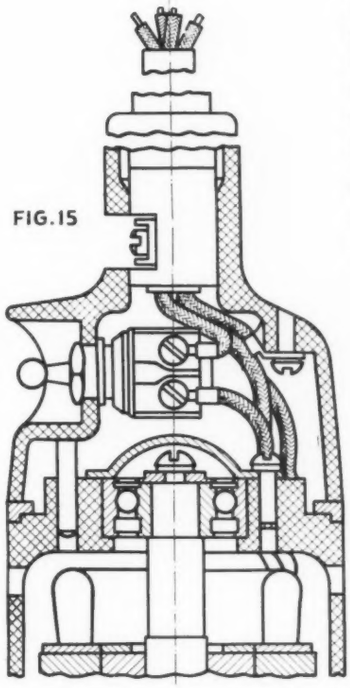


FIG. 15

Fig. 15—Latest design of switch arrangement for small die grinders and sanders. The toggle-operated switch unit is mounted in a dust-tight compartment and a dust-seal is provided where the toggle comes through the case. This type of switch does not have a release arrangement that opens it automatically.

Fig. 16—This switch is of the same type as shown in Fig. 11. It is mounted in a longer handle, being actuated by a remote trigger arrangement. It is provided with an additional return spring for quick action and also has a locking pin for holding the switch in the closed position when the trigger is released. A slight pull on the trigger releases the locking pin and opens the switch.

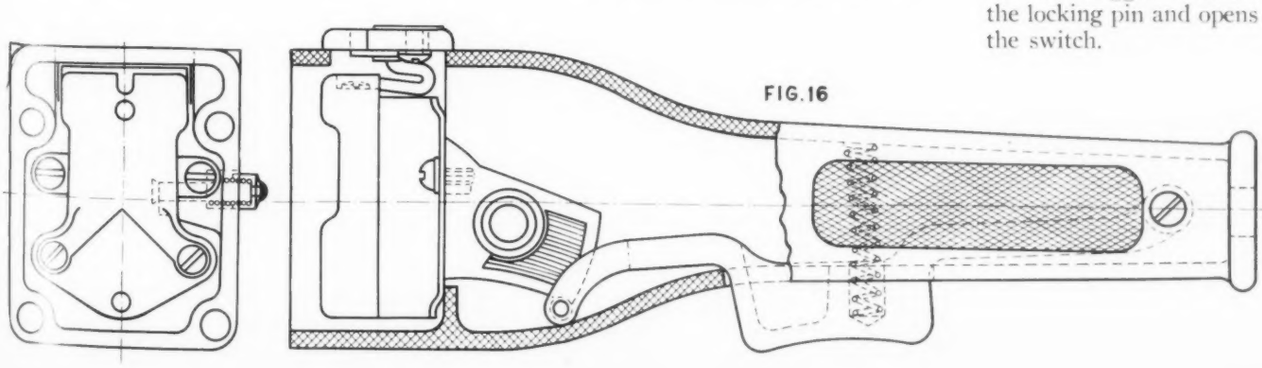
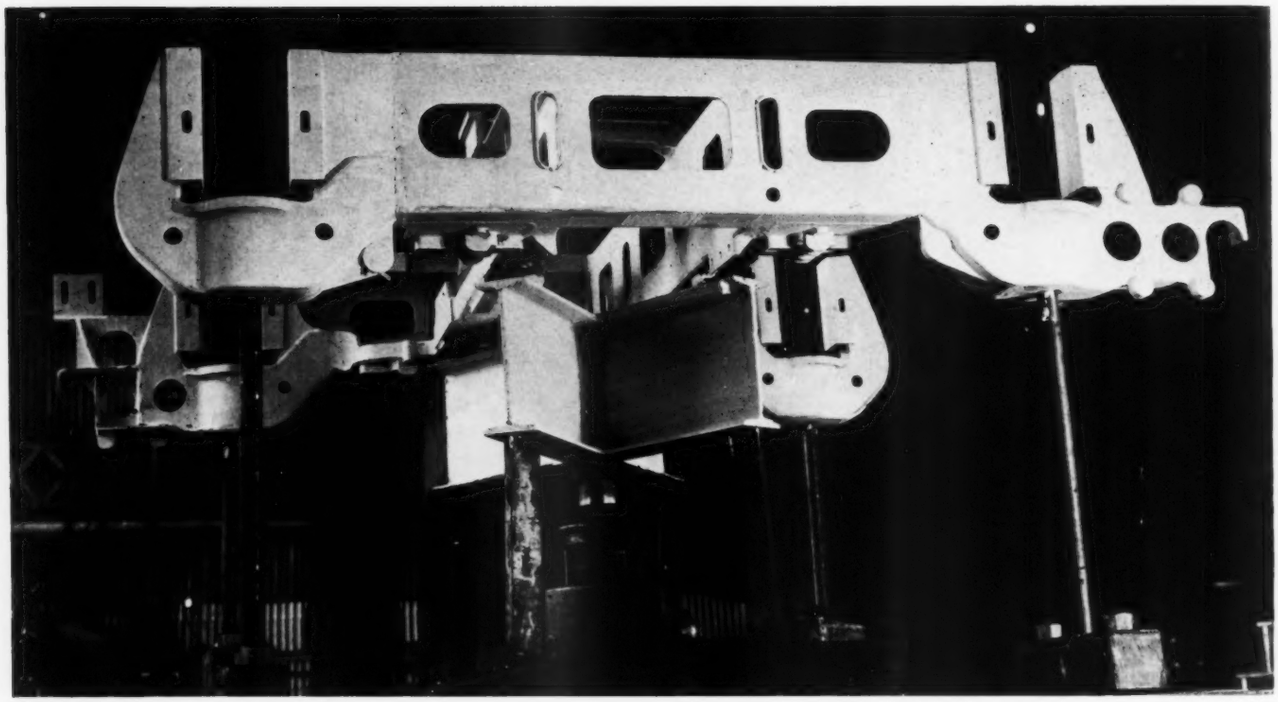


FIG. 16

NEWS

Welded Steel Trucks for Light-Weight Cars



Bottom view of set-up used in destruction test of welded steel truck frames for the B. M. T. five-car articulated train

ONE of the multi-section cars built in 1934 for the B.M.T. Lines of New York by the Edward G. Budd Manufacturing Company was equipped with welded steel truck frames, bolsters and spring planks. These were fabricated by Lukenweld, Inc. Much experience was gained in the design and operation of these trucks, with the result that the same type of construction is now being used for the trucks for the ten five-car multi-section units now being built by the St. Louis Car Company for the B.M.T. Lines.

The accompanying illustration shows the set-up for one of the tests made on these trucks for the purpose of determining the over-all strength of the truck frame itself. According to the report of this test submitted by Lukenweld, after loading in increments of 5 tons up to 85 tons, no permanent set was discernable after releasing the load. At 99 tons failure occurred by crippling of the thin unsupported transom web at the point where the transom intersects the side member.

In the new design of the welded truck, one important change was the introduction of single shoe brakes. The two brake cylinders were located at the extreme ends of the truck, diagonally opposite. The pedestals are of cast steel, Cromansil, butt-welded to the $\frac{1}{4}$ in. formed side member section at a point immediately behind the inner journal box guides.

At a center plate loading of 34,000 lb. the design stress in the trucks is approximately 24,000 lb. per sq.in., which is about 40 per cent of the endurance limit of the material. This may seem to be high, but it is the sum of the stresses resulting from all possible loads that might occur simultaneously. Indeed, in the case of the Burlington three-car train, the "Zephyr," the calculated maximum stress is 26,000 lb. per sq.in., and the successful service of this equipment over 3 years indicates that the figure is not too high.

Weight reduction was one of the principal aims in the design. The welded truck frame weighs 1,435 lb. as shipped, the welded steel spring planks weigh 80 lb. each, and the welded bolster weighed about 340 lb. before machining.

Properties of Sprayed Molten Metal

BECAUSE of the low tensile strength and brittle characteristics of sprayed metal, coatings should never be applied if there will be any fabrication such as bending or rolling of the material after it has been sprayed. Although the compressive strength of the material is high, its tensile strength is low and the material is brittle primarily because of two factors. This was brought out in a paper, "The Spraying of Molten Metal," by L. E. Kunkler, president of Metallizing Company of America, presented before the Spring Meeting of the American Society of Mechanical Engineers at Dallas, Texas on June 17. Mr. Kunkler explained that because of the manner in which the

molten metal is applied there is only a mechanical bond. Furthermore, an oxide is introduced by the spraying. This last is aggravated when successive layers are built up. By putting all of the metal on in one traverse, a large percentage of the oxide can be eliminated.

According to Mr. Kunkler, much emphasis must be placed on the preparation of the surface. The primary requirements are that the surface should be cleaned and roughened by a gritty sand blast, using sharp grits that will tear and make a jagged surface to which the metal can hold. Dove-tailing by machining is another method and rough-threading is a third. In every instance it is necessary that the surface be rough in order to establish a mechanical bond between the sprayed and parent metal.

According to the author, the approximate hardness of various sprayed metals is as follows: bronze 17; phosphor bronze, 6; 0.10 carbon steel, 26; 0.40 carbon steel, 40; 0.80 carbon steel, 60; 1.20 carbon steel, 75 sclerescopic hardness.

With reference to the bond strength of sprayed molten metal it seems to be about 700 lb. per sq.in. on flat surfaces.

Outstanding Papers at Pittsburgh Meeting

UNDER the auspices of the Pittsburgh Section of the American Society of Mechanical Engineers and the Carnegie Institute of Technology, the Fourth National Applied Mechanics Meeting was held June 11-13 at the Carnegie Institute of Technology, Pittsburgh, Pa. The technical program consisted of four sessions.

At the session on thin wall structures, Dr. J. N. Goodier of Toronto, Ont., delivered a paper on the "Thermal Stresses in Thin Cylinders Due to Temperature Variation Round the Circumference and Through the Wall." Other papers at this same session were "Buckling of Compressed Rectangular Plates with Built-in Edges," by Dr. J. L. Maulbetsch of the University of Michigan, and "The Behavior of Rectangular Flat Plates Under Concentrated Loads" by R. J. Sturm and R. L. Moore of the Research Laboratories of the Aluminum Company of America. In addition, the following papers were presented by title: "Cantilever Plate with Concentrated Edge Load," by Dr. D. L. Holl of the University of Iowa; "The Torsionless Bending of a Hollow Beam by a Transverse Load" by Professor W. L. Schwalbe, of the University of Illinois; "Stability of Rectangular Plates Under Shear and Bending Forces," by Dr. S. Way of the Research Laboratories of the Westinghouse Electric & Manufacturing Company.

At the session on plastic flow and creep, S. C. Hollister of Cornell University presented a paper "Plastically Pre-stressing the Region about a Hole in a Plate." At this same session was an informal discussion of the problem of creep under combined stress.

At the third session which was devoted to the subject of vibration, C. Concordia and A. L. Kimball of the General Electric Company presented a paper, "Vibration Reduction at Critical Speeds." At the same session Dr. J. P. Den Hartog and R. M. Heiles of Harvard University, presented a paper on "Forced Vibration in Non-Linear Systems with Various Combinations of Linear Springs," and S. J. Mikina and J. G. Baker of the Research Laboratories of the Westinghouse Electric & Manufacturing Company presented a paper on "Criteria for Stability of Vibrating Systems."

The fourth session was devoted to general mechanics. The following papers were presented: "An Electrical Resistance Method of Determining the Mean Surface Temperature of Tubes" by John H. Marchant, Columbia University; "An Analysis of Journal Bearings of Finite Length," by Professor E. O. Waters, Yale University; and "An Approximate Method for the Determination of Principal Stresses" by Professor Max M. Frocht of Carnegie Institute of Technology.

Do You Know That—

• Cellulose film, now being used for wire insulation on electric motors and household electric cords has a unique advantage. Printing on the cellulose film which is wrapped next to the wire serves as a permanent oil and stain proof identification of the particular make and year of manufacture of the wire. (19)

• A recently perfected process has made available both aluminum or steel rolls clad with Monel metal or nickel. This is not a plating or spraying process, but is analogous to nickel cladding as on flat sheets. (20)

• By a new process stainless steel can be given any color finish desired. It is explained that by treating the oxide film on the surface by chemical and thermal action, the proper color will be reflected. Said to withstand ordinary fabricating operations and to give increased resistance to acids. (21)

• Glass has been formed into helical coils which possess the properties of a spring. Such coils, approximately ¼ in. in diameter and of glass "wire" slightly less than 1/16 in. in diameter, have a deflection of about 20 per cent of the free length. The springs have been used experimentally where resistance to corrosion was required. (22)

• Magnetic chucks using permanent magnets instead of electro magnets are now available. By ingenious design, a pull of about 150 lb. per sq.in. is obtained for chucking. (23)

(For details, write the editor, giving the item number.)

EXHIBITIONS

Great Lakes Exposition — Cleveland, Ohio, June 29-October 4. Lincoln G. Dickey, manager, Terminal Tower, Cleveland, Ohio.

National Electrical and Radio Exposition—Madison Square Garden, Sept. 18-28. Joseph Bernhard, managing director, Grand Central Palace, New York, N. Y.

National Metals Congress & Exposition — 1936 Annual meeting, Cleveland, Ohio, Oct. 19-23. W. H. Eisenman, secretary, 7016 Euclid Ave., Cleveland, Ohio. The following national societies will participate: American Welding Society, the Wire Association, the Institute of Metals, the American Society for Metals, the American Institute of Mining & Metallurgical Engineers, and the American Society of Mechanical Engineers.

Brass Fittings Now Standardized

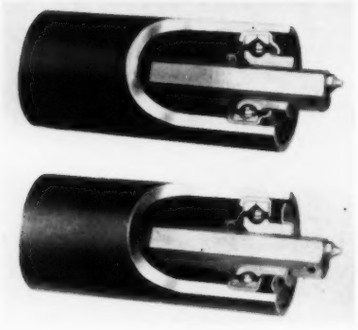
THE American Standards Association, 29 W. 39th St., New York City, announces the availability of printed copies of the American Standard Brass Fittings for Flared Copper Tubes (A40.2—1936) at 35 cents per copy. This standard resulted from the early work on brass fittings and flared copper water tubes carried on by a subcommittee of the Copper Tube & Fittings Manufacturers Standardization Committee. These new standards have been approved by the sectional committee of Sub-committee 7 on Brass Fittings for Copper Tubes of the ASA and the organizations directing the work of the committee, namely, the American Society of Mechanical Engineers and the American Society of Sanitary Engineering.

Now that the Standards for fittings have been approved, it is expected that the sectional committee will start work on standards for soldered joints. This increased activity is largely brought about by the greater interest in standardization of joints for copper tubing as a result of the marked growth in its use during the last few years.

New Materials and Parts

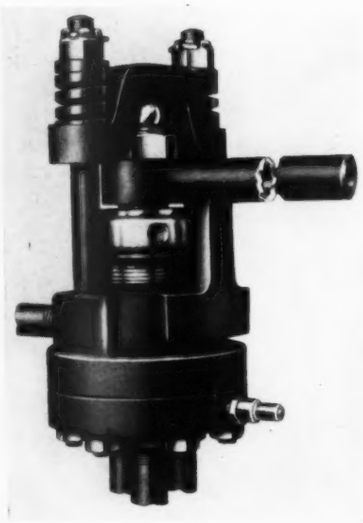
Non-Ferrous Expansion Plugs

Designed to close core holes and holes bored in castings of brass, copper, aluminum, stainless steel, monel metal. Since they are assembled so as to make a waterproof and air-tight seal, the co-efficients of expansion of the plug and casting should be the same and preferably of the same material. Wrought Washer Mfg. Co., 2100 S. Bay St., Milwaukee, Wis.



Protected Bearings

Four styles of gravity conveyor rolls, differing only in details, arranged to prevent the entrance of injurious material. In one type, felt washers seal the gaps between the inner and outer races at the front and the back to prevent grease dripping. Another type has a felt washer at the rear and a steel shield in front which will allow old grease to escape when refilling. Where heat is present an all-steel type can be used. A fourth type has a stationary flared outer shield to protect the bearings from molten metal. Standard Conveyor Co., North St. Paul, Minn.

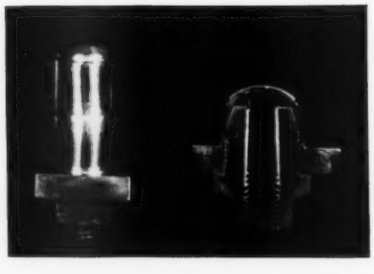


Hydraulic Valve

Balanced flat disk style for oil or water service. Can be used in any position. Pressure is always downward on the disk and upward on the adjustable spring counter-balanced stem. Drilled holes in the stem trap air when valve is filling to cushion water hammer shock. Furnished for pressures up to 5,000 lb. per sq.in., in 2, 3 and 4-way types, and in pipe sizes $\frac{1}{4}$ to 2 in., with cast steel or bronze body. W. H. Nicholson & Co., 12 Oregon St., Wilkes-Barre, Pa.

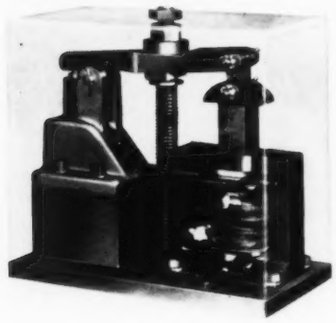
Mechanical Rivet

One-piece steel body having a square head at one end and a taper valve seat at the other. Steel expansion and compression ferrule fits on valve seat, a hardened steel bolt passes through the assembly. The head of the bolt fits against the ferrule. When nut is turned the ferrule is drawn up, expanded, and compresses the parts being riveted. Available in sizes for holes $\frac{1}{4}$ in. to 1 in. diameter. Multi-Seal Mfg. Co., 123 N. Jefferson St., Chicago, Ill.

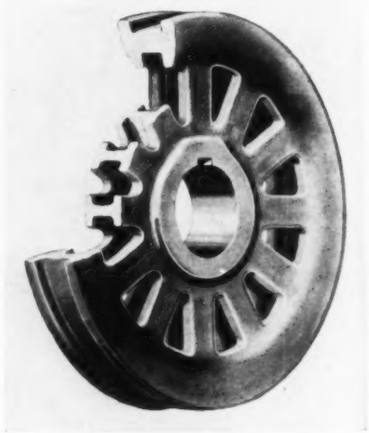


Solenoid Controlled Air Valve

Electrically actuated for operating pneumatic plunger and piston elements, for relieving pressure on tanks, and controlling signal alarms. Designed for severe shop service with remote controls and automatic circuit contacting devices. Can be furnished for either normally open or normally closed service. Ross Operating Valve Co., Detroit, Mich.



Double-Wall Crane Wheels



To eliminate the possible breakage in single web crane wheels, a double-wall wheel is now available. The walls are continuous with the flanges, providing a strong but elastic support and resistance to side thrusts. These walls are tied together with cross spokes of uniform I-section. The wheels are made in all diameters and for all cranes, with hub lengths, bores and other dimensions to suit the users' requirements. American Manganese Steel Co., Chicago Height Ill.

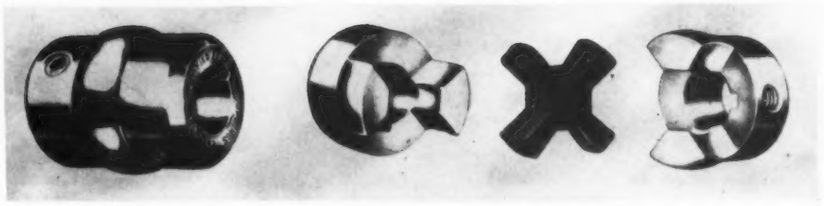
Suction Pumps

Employ a half journal bearing on the rotor to accommodate automatic takeup of lateral wear for improvement of vacuum. Furnished to oil burner manufacturers, in capacities from 15 to 60 gallons per hour, at a speed of 1,750 r.p.m. Capable of producing in excess of 300 lb. pressure. The pump shown in the upper part of the illustration is known as the XW-Type, and the one shown at the bottom is known as the YSL-Type. Mechanical Devices Co., Bloomington, Ill.

Single-Phase Induction Motor

For heavy industrial duty on such applications as pumps, compressors, machine tools, door openers and grease guns. The type CR motor starts as a

repulsion motor, developing a high torque. At approximately 60 per cent of synchronous speed, the armature is short circuited by a centrifugally operated device, and the motor then operates as a squirrel cage induction motor. Available in ratings, ¾ to 3 hp., 110/220 volts, dual voltage operation on all commercial frequencies. Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

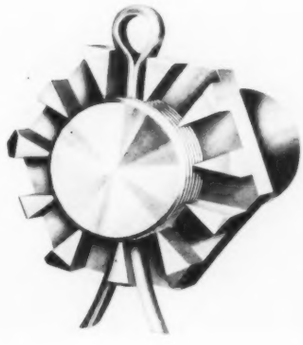


Flexible Coupling

Oil-resistant rubber intermediate part provides flexibility and resilience. Three simple members assure easy installation and alignment. It is claimed to have low flywheel effect. Made in three sizes, ½, 1 and 5-hp. ratings at 1,750 r.p.m. Available in die-cast finish and also chromium plated. Certified Flexible Couplings, Inc., 122 East 42d St., New York, N. Y.

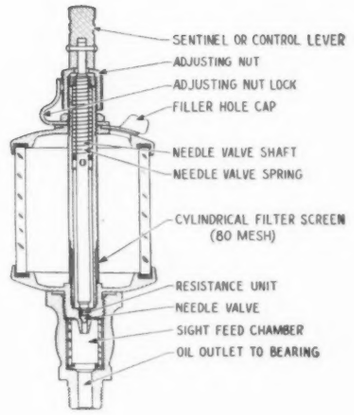
Micro Slotted Nut

Is used where closer adjustments are required than the usual six per turn of a hexagonal nut. This Cooke type of nut has 10 to 22 adjustments, depending on the nut size. Fine adjustment is accomplished by placing the keying wedges directly opposite from the slot that accommodates the cotter pin head. In the keying position the wedge lines up with the center of the pin head. With this closer adjustment the nut need not be backed off to obtain a keying position. The pin is set by simply tapping it with a hammer and spreads automatically, filling the hole. A special pin with internal bevelled edges accompanies each nut. The Blatchford Corp., 80 E. Jackson Blvd., Chicago, Ill.



Microflow Oil Cup

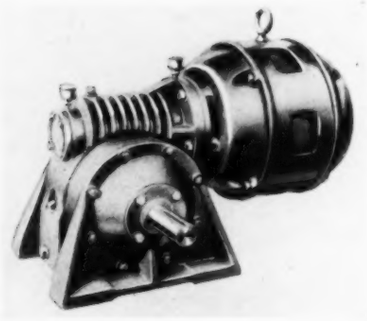
Is used for bearings requiring a continuous flow of oil in small, adjustable quantities. A resistance unit is incorporated which is said to permit five adjustments and to assume positive operation with no clogging. The oil is strained by an 80-mesh filter screen. It is also said that no dirt can enter the filler hole, yet enough



air is admitted to provide adequate pressure on the oil for feeding. The cup is chrome-plated. Alemite Division of Stewart-Warner Corp., 1826-52 Diversey Parkway, Chicago, Ill.

Interchangeable Control Units

Individual pilot light, transfer switch, and pushbutton units for any desired combination can be assembled in gray enameled boxes. Suitable for machine work and medium service jobs. Capacity of all units, 600 volts. Metal parts of units are cadmium plated to resist oxidation. Housings are available in steel, cast iron, flush with brass plate and box, or separate units for special control panel mounting. Arrow-Hart & Hegeman Electric Co., Hartford, Conn.

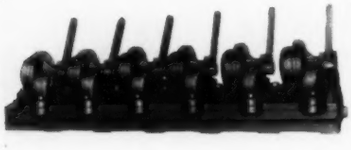


Motorized Speed Reducer

Two types, one a single worm and wormwheel reduction, the other a double worm gear reduction. In each, the worm gear drives a planetary spur gear unit. It is stated that this reducer has efficiencies comparable to spur gearing. In the first reduction a hardened alloy steel worm is used with a cast bronze gear. A pinion in mesh with three planet gears, which turn on hardened studs mounted on a disk on countershaft make up the second reduction. Sizes 1/50 to 5 hp. Janette Mfg. Co., 556-558 W. Monroe St., Chicago, Ill.

Manifold Valve

For control of compressed air or hydraulic equipment. Claims are made for use with fluid and gas handling operations in chemical and refinery processes. Inlet and exhaust connec-



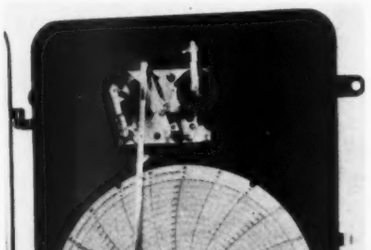
tions can be made at either end, cylinder ports in the bottom. The valves are packless. A ground and lapped bronze disk makes a seal with a similarly finished seat. Manifolds can be furnished with any number of valves in 3-way and 4-way types. Hannifin Mfg. Co., 621-630 S. Kolmar Ave., Chicago, Ill.

Plastic Molding Material

Durez 1544 combines machining qualities with high impact strength. Developed especially for molded parts such as telephone handles, and machine parts requiring higher than ordinary strength plus machinability. It is said the impact strength has been increased to almost twice that of regular phenolic materials, and the finished moldings can be machined, sanded, buffed or wire-brushed without uncovering filler spots. Subsequent buffing of the machined areas will bring up a black finish of unusual smoothness and luster. The weight of the material is 22.4 grams per cu.in. General Plastics, Inc., North Tonawanda, N. Y.

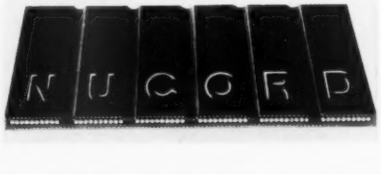
Recorder-Controller

New series now available for automatically controlling temperature, time-temperature, flow, liquid level, pressure, time-pressure, and humidity. Known as the Ampliset Free-Vane Controller. Features of field adjustability have been added whereby its sensitivity can be changed with a small screwdriver to suit lag. Can also be changed from direct-acting to reverse-acting or vice versa. The Bristol Co., Waterbury, Conn.



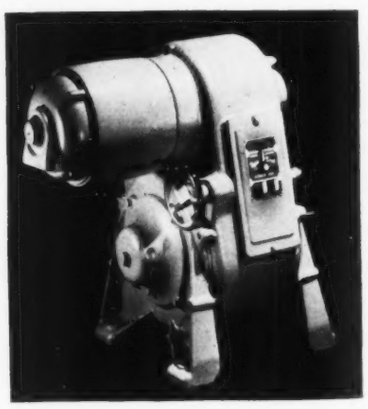
Multiple Flat Belt

The principle of "Nucord" multiple construction is that each cord section (between grooves) withstands an equal portion of the load. It is said that as the belt bends over the pulley, the grooves on the outside surface of the belt reverse themselves slightly, thereby producing vents for air trapped between belt and pulley to escape. The 10 gum-dipped cords of each section withstand the belt strain both longitudinally and laterally, each cord having tensile strength of more than 100 lb. The treated fabric around the cords is said to protect the cords against deterioration from heat, water, steam, oil or acid fumes. It is also said to have sufficient elasticity and resiliency to eliminate stretch or permanent set. When made endless the cords are so laid that there are no splices. Endless belts are furnished in minimum lengths of 37 in. for $\frac{3}{4}$ to $1\frac{3}{4}$ in. wide and in 64-in. lengths for belts 2 to 12 in. wide. Maximum lengths of endless belts can be had up to 300 ft. Roll belting is furnished in lengths of 500 ft. for any width from $\frac{3}{4}$ to 12 in. Any type of metal fastener or rawhide can be used for joining. Nucord Co., 605 West Washington Blvd., Chicago, Ill.



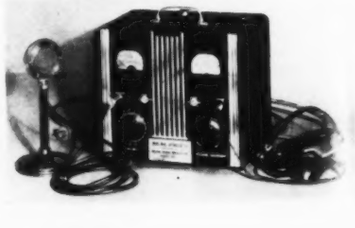
Speed-Trol Power Units

Infinitely variable speed V-belt transmission built together with motor. For industrial processes requiring speed change. Motors can be furnished with reduction gears. The unit can be mounted with shaft vertical or horizontal; feet can be left off for built-in applications. Remote, automatic, and differential controls are available. Totally inclosed. Standard speed variation from 2:1 to 6:1, and capacities of $\frac{1}{4}$ to 15 hp. ratings. Sterling Electric Motors, Inc., Los Angeles, Calif.

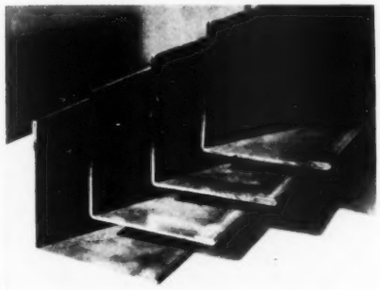


Sound Level Indicator

Device for measuring intensity and character of sounds in studying general and industrial noise problems. Consists of six major divisions, a calibrated microphone, an audio-frequency amplifier, a calibrated attenuator, a weighing filter, a calibrated decibel meter, and a head set. The



model 300 indicator has a reference level of 10^{-16} watts; the basis of calibration is 1,000 cycles per second. Has a range from 20 to 102 decibels. Operates on 110 volt, 60 cycle power supply. Battery operated units are also available. The complete apparatus weighs 25 lb. The size of the cabinet is 12x10x8 in. Industrial Apparatus Co., 720 N. Wabash Ave., Chicago, Ill.



High-Tensile Steel Sheets

Tests on specimens of Armco H.T. 50 are said to show an impact tensile strength of 5,000 ft.-lb. per sq.in., or nearly double that of mild steel, permitting the use of advanced methods of fabrication and welding. In this material the endurance limit is 48,000 lb. per sq.in. Tensile strength is 67,000 lb. per sq.in. in hot rolled grades with elongation of 28 per cent. Cold rolled grades have a tensile strength of 70,000 lb. per sq.in. Atmospheric corrosion resistance is said to be from four to six times that of ordinary steel. Sheets and plates are available in 20-gage and heavier and in all finishes. The American Rolling Mill Co., Middletown, Ohio.

Metal-Bord

A pre-finished metal sheet cemented to a non-metallic backing. Used for buckles, dress ornaments, advertising specialties, display devices and wherever a metal surface on a non-metallic base is desired. Adapts itself to blanking and shearing machinery. Permits using a light gage metal but is strong and flexible. Available in plain, striped, and crimped patterns, and thickness to suit requirements, in sheets up to 36x96 in. American Nickeloid Co., Peru, Ill.

Fulscope Indicating Controllers

For processes requiring close throttling control, is operated by air. Simple adjustment can be made to adapt controller to any changes in

processing requirements. Made for either high range or full range sensitivity. The control point may be adjusted to any value within nominal range. Controller action can be reversed in the field by altering the position of a link. Temperature controllers may be equipped with mercury, vapor or gas-actuated tube systems. Both face and flush mounting styles are available in black finished cast aluminum case. Taylor Instrument Companies, Rochester, N. Y.

Black Finish Process

Applied to articles fabricated from iron or steel. The Jetal process of oxidizing gives a uniform jet black color by simple immersion in an aqueous bath, prepared by dissolving Jetal salts in water and heating to about 270 deg. F. The work to be colored must be previously cleaned in a pickling bath. It is stated that the lustrous finish is a part of the metal, and cannot chip, scale, peel, or discolor, and that after application the metal can be stamped and rolled without injury to the finish. Alrose Chemical Co., 80 Clifford St., Providence, R. I.

Rubber Novelty Matting

Used for pads on scales and stepladders, tops for counters and display tables. Known as grade 5075, it is available in black, either 36 or 48 in. wide, about $\frac{3}{32}$ in. thick, and is sold in 10-yard rolls. B. F. Goodrich Co., Akron, Ohio.

MANUFACTURERS' PUBLICATIONS

Ampco Guide Bushings — Ampco Metal, Inc., Milwaukee, Wis. Data Sheet No. 29, 8½x11 in. Table of dimensions of standard Ampco metal centrifugal cast guide pin shoulder bushings, with a description of their advantages when used for die guide bushings.

Ampliset Free-Vane Controllers — The Bristol Co., Waterbury, Conn. Bulletin No. 440, 8 pages, 8x10½ in. Describes a new series of controllers which are available for time-temperature, flow, liquid level, pressure, time-pressure and humidity, operating on the free-vane principle of pneumatic control. Illustrated with half tones, diagrams and charts.

Bearings for Electric Motors—Johnson Bronze Co., New Castle, Pa. Catalog EM-6, 32 pages, 8½x11 in. Features complete line of bearings with comprehensive alphabetical, numerical, and progressive listings for easy reference. Individual bearings are illustrated. Of especial interest to the electric motor bearing trade. Cast and machined bars for general purpose bearings are also listed.

Chromel Resistance Wire—Hoskins Mfg. Co., Detroit, Mich. Catalog K. 48 pages, 5⅞x7½ in. Data covering sizes, weights, and electrical properties, of resistance wire, ribbon, rounds, squares and flats, with design information for heating units and elements.

Chain Data — Columbus McKinnon Chain Corp., Tonawanda, N. Y. Catalog, Issue No. 7, "Useful Information on Welded and Weldless Chain," 52 pages, 8½x11 in. Tables giving Trade sizes and actual sizes, weights, test loads and safe working loads of various kinds of chain and hooks. Slings, shackles, and chain dogs are also listed.

Conveyor and Elevators—Stephens-Adamson Mfg. Co., Aurora, Ill. Catalog 42, Redler Conveyor & Elevators, 36 pages, 8½x11 in. Describes and illustrates types of unit driving, feeding, and discharging equipment and complete systems for handling bulk materials. The catalog is appropriately illustrated with installation photographs and line drawings. It also contains tables of dimensions and design data.

Drum Controllers—Furnas Electric Co., West Allis, Wis. Catalog 36, 12 pages, 8½x11 in. General description of design and construction, with illustrations, line cuts, dimensions, and wiring diagrams, of foot-operated switches, reversing drum switches and speed controllers. A few industrial applications are shown.

Properties of Silmo Steel — Timken Steel & Tube Co., Canton, Ohio. 6 loose leaf pages. Physical data on "SILMO" steel is presented covering high temperature tensile properties, Charpy value at elevated temperatures, heat embrittlement tests, creep strength, temperature-stress embrittlement tests, stability, and corrosion and oxidation resistance.

Regulating Input to Demand—Leeds & Northrup Co., 4934 Stenton Ave., Philadelphia, Pa. Catalog N-OOA, "Micromax Electric Control," 8 pages,

7¾x10½ in. Describes a complete control system, for proportioning input to demand based on a simple potentiometer balance. The apparatus consists of three integral parts, a Micromax control instrument, a relay detector and a valve mechanism, to form a sensitive electrical balance. Any fluctuation in the condition being controlled results in a correcting movement of input valve.

Power Transmission Equipment — Link-Belt Co., 2410 W. 18th St., Chicago, Ill. Catalog No. 1500, 208 pages, 6x9 in. Devoted exclusively to power transmission equipment, and features the recently announced lines of anti-friction roller bearing units, streamlined babbitted bearings, and new types of take-ups. Hangers, pulleys of all types, gears, shafting, couplings, collars, and grease fittings are also included. Amply illustrated and supplied with dimensions, weight tables and design data.

Seamless Tubing—Parker Appliance Co., 17325 Euclid Ave., Cleveland, Ohio. Bulletin No. 41, Parker Seamless Tubing, 36 pages, 8½x11 in. Price list and technical data covering copper, 3½ per cent nickel, stainless iron, high tensile bronze, and special purpose tubing. Sizes, weights, general specifications, and safe internal working pressures are given in table form for handy reference.

Welding and Cutting High Chromium—Linde Air Products Co., 30 East 42d St., New York, N. Y. Bulletin, 8 pages, 8½x11 in. Effect of chromium on welding is discussed with recommendations for various types of high chromium steels. Characteristics resulting from additions of columbium to both base metal and welding rod are described with handling technique.

Variable-Speed Transmission — Lewellen Mfg. Co., Columbus, Ind., Catalog No. 25-D, "Lewellen Variable-Speed Transmission and Controlling Devices," 48 pages, 8½x11 in. Describes construction, principles of operation, lubrication, motor mountings, shifting devices and indicators. The section on controls covers mechanical, electrical, remote, and automatic types. Special applications are enumerated, with recommendations for installation. Dimension tables and design data are included. Illustrated with about 150 halftones not including numerous line cuts.

BOOKS AND BULLETINS

S.A.E. Handbook

1936 Edition. 776 pages, 5½x8⅜-in. Blue leatherette covers. Published by the Society of Automotive Engineers, Inc., 29 West 39th St., New York, N. Y. Price $5 to non-members.

This comprehensive annual publication contains all current standard specifications and recommended practices adopted by the society. New specifications are included for storage battery life tests, electric lamp bulbs for aircraft, tube fittings with modified thread, and bearing bronze S.A.E. No. 660. Woodruff key slot cutters and gages have been added. Revisions are listed affecting motor-vehicle storage batteries, electric lamp bulbs for motor vehicles and motorboats, ball bearing locknuts and washers, fuel and tube fittings, and twist drills. Specifications have been corrected for fuses and fuse clips, also woodruff key slots and keyways. Cancellations of previous standards are stated. Officers of the society and personnel of all the committees and divisions are listed.

The handbook is made up of ten general sections, and has a good index of contents. Subject matter deals with dimensional standards for parts used in automotive apparatus; steel, iron, and non-ferrous metal specifications and classifications; lubricant viscosity numbers and fuel rating. A fabricated materials section includes structural tubing, extruded shapes, leather, felt, plate glass, conduit and insulated cable. Four sections are devoted to tests, ratings, codes, transportation, maintenance, tools, jigs, gages, nomenclature, and definitions.

The book is a standard reference volume for engineers and designers engaged in activities allied with the automotive industries.

Handbook of Chemistry and Physics

Charles D. Hodgman. 1,966 pages, 4½x6¾ in. Simulated leather covers. Published by the Chemical Rubber Publishing Co., Cleveland, Ohio. Price $6.

The handbook is a representation of 22 years accumulation of necessary data for the engineer and scientist. The first section of this latest edition is devoted to 270 pages of mathematical tables. The second section of 535 tables contains properties and physical constants of the elements, organic and inorganic compounds and materials. Over 1,000 organic compounds have been added making a list of nearly 9,000. There are 364 pages of general chemical tables and specific gravity and properties of matter. Another section of 518 pages treats on heat and hygrometry, sound, electricity and magnetism, and light. The last section with 312 pages relates to quantities and units, and miscellaneous tables.

This new edition has more than 300 pages of new composition and a number of tables have been completely revised and rewritten. The section on colorimetry and spectrophotometry has added material. The section on X-ray spectra has been revised to conform to the results of more recent determination.

Handbook of Welded Steel Tubing

Formed Steel Tube Institute. Loose leaf book, 60 pages, 5½x8⅜ in. Distributed by Steel & Tubes, Inc., 224 East 131st St., Cleveland, Ohio.

The purpose of this handbook is to give the user a knowledge of the application of welded steel tubing. It contains data to guide engineers and laymen. Descriptions of welding processes are given, with notes on selection of material, and tables of tolerances. A table of critical speeds of welded tubular propeller shafts is inserted. Physical and chemical data is supplied. Recommended dimensions of flanged, expanded, swaged, and formed tubing are tabled.

• • •

Fabrication and Treatment of USS Stainless Steels

Peter A. Frasse & Co., Inc., 17 Grand St., New York, N. Y. Booklet, 42 pages, 4¾x7¾ in. Paper cover.

The publication contains a summary of information on the fabrication and treatment of USS stainless and heat-resisting steels. It deals with welding, finishing, soldering, brazing and riveting. Contains sections on annealing, removal of oxide and scale, machining and cutting operations, drawing and forming, and finishing of fabricated articles. It also takes up the bending, spinning, swaging and machining of stainless and heat-resisting steel tubes. Includes tables of bending radii for tubing and pipe, standard types and sizes of USS steel with analysis data, and nominal properties.

The book is of interest not only for the general information presented but because of the practical help it contains for engineers and shop foremen.

A Century of Machine Tools

The Taylor & Fenn Co., 54 Arch St., Hartford, Conn. Booklet, 22 pages, 6x9 in. Paper cover.

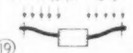
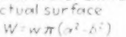
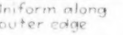
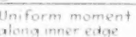
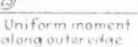
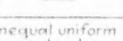
As the title implies, this booklet contains a resume of machine tool design and manufacture by this company from 1834 to the present time. It will be of interest to engineers and designers to see pictures of drilling machines, lathes, punch presses and gear cutting machines made as early as 1840. It was in this plant that the Lincoln type of milling machine originated. There are in the company's show rooms some of these old machines, and in the shop, still under power, is a large engine lathe for machining locomotive driving wheels. The machine has a 84 in. swing and is 7 ft. between cente

Stresses and Deflections in Flat Plates
RAYMOND J. ROARK

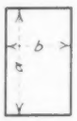
Stresses and Deflections in Flat Plates

RAYMOND J. ROARK

(continued from page 254)

Circumstances and Reference Number			STRESS	DEFLECTION
Form of Plate	Manner of Support	Manner of Loading and Reference No.		
Circular with concentric hole (Circular flange)	Outer edge fixed and supported	Uniform along inner edge (18)	At outer edge $\text{Max. } S_r = \frac{3W}{2\pi t^2}\left[1-\frac{2mb^2-8b^2(m+1)\log\frac{a}{b}}{a^2(m-1)+b^2(m+1)}\right]$ At inner edge $\text{Max. } S_t = \frac{3W}{2\pi mt^2}\left[1+\frac{ma^2(m-1)-mb^2(m+1)-2(m^2-1)a^2\log\frac{a}{b}}{a^2(m-1)+b^2(m+1)}\right] = \text{Max. } S_y$	At inner edge $\text{Max. } y = -\frac{3W(m^2-1)}{4\pi m^3Et^3}\left\{a^2-b^2-\frac{2mb^2(a^2-b^2)+\left[6b^4(m+1)-2a^2b^2(7m+3)\right]\log\frac{a}{b}+4a^2b^2(m+1)(\log\frac{a}{b})^2}{a^2(m-1)+b^2(m+1)}\right\}$
	Outer edge fixed and supported, inner edge fixed	Uniform over entire actual surface $W=w\pi(a^2-b^2)$  (19)	At outer edge $\text{Max. } S_r = \frac{3w}{4t^2}\left[(a^2-3b^2)+\frac{4b^4}{a^2-b^2}(\log\frac{a}{b})\right]$ At inner edge $S_r = -\frac{3w}{4t^2}\left[(a^2+b^2)-\frac{4a^2b^2}{a^2-b^2}(\log\frac{a}{b})\right]$	At inner edge $\text{Max. } y = -\frac{3w(m^2-1)}{16m^2Et^3}\left(a^4+3b^4-4a^2b^2-4a^2b^2\log\frac{a}{b}\right)$
		Uniform along inner edge (20)	At outer edge $S_r = \frac{3W}{2\pi t^2}\left[1-\frac{2b^2}{a^2-b^2}(\log\frac{a}{b})\right]$ At inner edge $\text{Max. } S_r = \frac{3W}{2\pi t^2}\left[1-\frac{2a^2}{a^2-b^2}(\log\frac{a}{b})\right]$	At inner edge $\text{Max. } y = -\frac{3W(m^2-1)}{4\pi m^3Et^3}\left[a^2-b^2-\frac{4a^2b^2}{a^2-b^2}(\log\frac{a}{b})^2\right]$
	Inner edge fixed and supported	Uniform over entire actual surface $W=w\pi(a^2-b^2)$  (21)	At inner edge $\text{Max. } S_r = \frac{3w}{4t^2}\left[\frac{4a^4(m+1)\log\frac{a}{b}-a^4(m+3)+b^4(m-1)+4a^2b^2}{a^2(m+1)+b^2(m-1)}\right]$	At outer edge $\text{Max. } y = -\frac{3w(m^2-1)}{16m^2Et^3}\left[\frac{a^6(7m+3)+b^6(m-1)-a^4b^2(m+7)-a^2b^4(7m-5)-4a^2b^2\left[a^2(5m-1)+b^2(m+1)\right]\log\frac{a}{b}-16a^4b^2(m+1)(\log\frac{a}{b})^2}{a^2(m+1)+b^2(m-1)}\right]$
		Uniform along outer edge  (22)	At inner edge $\text{Max. } S_r = \frac{3W}{2\pi t^2}\left[\frac{2a^2(m+1)\log\frac{a}{b}+a^2(m-1)-b^2(m-1)}{a^2(m+1)+b^2(m-1)}\right]$	At outer edge $\text{Max. } y = -\frac{3W(m^2-1)}{4\pi m^3Et^3}\left[\frac{a^4(3m+1)-b^4(m-1)-2a^2b^2(m+1)-8ma^2b^2\log\frac{a}{b}-4a^2b^2(m+1)(\log\frac{a}{b})^2}{a^2(m+1)+b^2(m-1)}\right]$
	Outer edge fixed	Uniform moment along inner edge  (23)	At inner edge $\text{Max. } S_r = -\frac{6M}{t^2}$ At outer edge $S_r = -\frac{6M}{t^2}\left[\frac{2mb^2}{(m+1)b^2+(m-1)a^2}\right]$	At inner edge $\text{Max. } y = \frac{6M(m^2-1)}{mEt^3}\left[\frac{a^2b^2-b^4-2a^2b^2\log\frac{a}{b}}{a^2(m-1)+b^2(m+1)}\right]$
	Inner edge fixed	Uniform moment along outer edge  (24)	At inner edge $\text{Max. } S_r = -\frac{6M}{t^2}\left[\frac{2ma^2}{(m+1)a^2+(m-1)b^2}\right]$ At outer edge $S_r = -\frac{6M}{t^2}$	At outer edge $\text{Max. } y = \frac{6M(m^2-1)}{mEt^3}\left[\frac{a^4-b^2-2a^2b^2\log\frac{a}{b}}{a^2(m+1)+b^2(m-1)}\right]$
	Outer edge supported	Unequal uniform moments along edges  (25)	At a $S_r = -\frac{6}{t^2(a^2-b^2)}\left[a^2M_a-b^2M_b-\frac{a^2b^2}{r^2}(M_a-M_b)\right]$ $S_t = -\frac{6}{t^2(a^2-b^2)}\left[a^2M_a-b^2M_b+\frac{a^2b^2}{r^2}(M_a-M_b)\right]$	At a $y = \frac{12(m^2-1)}{mEt^3(a^2-b^2)}\left\{\frac{a^2-r^2}{2}\left[\frac{a^2M_a-b^2M_b}{m+1}\right]+\log\frac{a}{r}\left[\frac{a^2b^2(M_a-M_b)}{m-1}\right]\right\}$ (from outer edge level)
Elliptical solid $b/a=\alpha$	Edge supported	Uniform over entire surface (26)	At center (2) $S_b = -\frac{0.135(2-\alpha)wb^2}{t^2} = \text{Max. } S$	At center (2) $\text{Max. } y = -\frac{(0.146-0.1\alpha)wb^4}{Et^3}$
		Uniform over small concentric circular area of radius r_0 (27)	At center (2) $S_b = -\frac{3W}{2\pi mt^2}\left[\log\frac{b}{2r_0}(m+1)+6.57-2.57\alpha\right] = \text{Max. } S$	At center (2) $\text{Max. } y = -\frac{Wb^2}{Et^3}(0.19-0.07\alpha)$
	Edge fixed	Uniform over entire surface (28)	At edge (Span a) $S_a = \frac{3wb^2\alpha^2}{2t^2(3+2\alpha^2+3\alpha^4)}$ (Span b) $S_b = \frac{3wb^2}{2t^2(3+2\alpha^2+3\alpha^4)} = \text{Max. } S$ At center $S_a = -\frac{3wb^2\alpha^2}{4t^2(3+2\alpha^2+3\alpha^4)}$ $S_b = -\frac{3wb^2}{4t^2(3+2\alpha^2+3\alpha^4)}$	At center $\text{Max. } y = -\frac{3w(m^2-1)b^4}{16m^2Et^3(6+4\alpha^2+6\alpha^4)}$
		Uniform over small concentric circular area of radius r_0 (29)	At center (3) $S_b = -\frac{3W(m+1)}{2\pi mt^2}\left[\log\frac{b}{r_0}-0.317\alpha-0.376\right]$	At center (3) $\text{Max. } y = -\frac{(3\alpha^2)(0.845-0.026\alpha)}{Et^3}$

(continued on next page)

Circumstances and Reference Number			STRESS	DEFLECTION
Form of Plate	Manner of Support	Manner of Loading and Reference No.		
Square Solid	Edges supported above and below (corners held down)	Uniform over entire surface (30)	At center $S_a = -\frac{0.2208\,w\,a^2(m+1)}{m\,t^2} = \text{Max. } S$	At center (4) Max $y = -\frac{0.0443\,w a^4}{E t^3}$
		Uniform over small concentric circular area of radius r_0 (31)	At center (5) Max. $S = -\frac{3W}{2\pi m t^2}\left[\log\frac{a}{2r_0}(m+1) + 0.75\,m\right]$	At center (5) Max $y = -\frac{0.1391\,W a^2(m^2-1)}{m^2 E t^3}$
	Edges supported below only (corners free to rise)	Uniform over entire surface (32)	At center, on diagonal section $S = -\frac{0.2214\,w a^2}{t^2}$ At corners, on diagonal section $S = -\frac{0.277\,w a^2}{t^2} = \text{Max. } S$	
		Uniform over small concentric circular area of radius r_0 (33)	Same as Case 31	Same as Case 31
	All edges fixed	Uniform over entire surface (34)	At center of each edge $S_a = \frac{0.308\,w a^2}{t^2} = \text{Max. } S$ At center $S_a = -\frac{0.14\,w a^2}{t^2}$	At center Max $y = -\frac{0.0138\,w a^4}{E t^3}$
		Uniform over small concentric circular area of radius r_0 (35)	At center (6) Max $S = -\frac{3W}{2\pi m t^2}\left[\log\frac{a}{2r_0}(m+1)\right]$	
Rectangular Solid  $\frac{b}{a} = \alpha$	All edges supported	Uniform over entire surface (36)	At center (4) $S_a = -\frac{w b^2(0.285 + 0.382\,\alpha^2 - 0.320\,\alpha^3)}{t^2}$ (4) $S_b = -\frac{0.75\,w b^2}{t^2(1+1.61\,\alpha^3)} = \text{Max. } S$	At center (4) Max $y = -\frac{0.1422\,w b^4}{E t^3(1+2.21\,\alpha^3)}$
		Uniform over small concentric circular area of radius r_0 (37)	At center (5) $S_b = -\frac{3W}{2\pi m t^2}\left[\log\frac{b}{2r_0}(m+1) + m(1+K)\right]$ where $K = \left(\frac{0.814}{1+2.25\,\alpha^5}\right) - 0.5$	At center (5) Max $y = -\frac{0.203\,W b^2(m^2-1)}{m^2 E t^3(1+0.462\,\alpha^4)}$
	All edges fixed	Uniform over entire surface (38)	At centers of long edges (4) $S_b = \frac{0.5\,w b^2}{t^2(1+0.623\,\alpha^6)} = \text{Max. } S$ At center (8) $S_b = -\frac{0.75\,w b^2}{t^2(3+4\,\alpha^4)}$ At centers of short edges (8) $S_a = \frac{0.25\,w b^2}{t^2}$ (8) $S_a = -\frac{0.054\,w b^2(1+2\alpha^2-\alpha^4)}{t^2}$	At center (4) Max $y = -\frac{0.0284\,w b^4}{E t^3(1+1.056\,\alpha^5)}$
		Uniform over small concentric circular area of radius r_0 (39)	At center (7) $S_b = -\frac{3W}{2\pi m t^2}\left[\log\frac{b}{2r_0}(m+1) + 5(1-\alpha)\right] = \text{Max. } S$	
	Long edges fixed Short edges supported	Uniform over entire surface (40)	At centers of long edges (4) $S_b = \frac{w b^2}{2t^2(1+0.2\,\alpha^4)} = \text{Max. } S$ At center (4) $S_b = -\frac{w b^2}{4t^2(1+0.4\,\alpha^4)}$ (4) $S_a = -\frac{3 w b^2(1+0.3\,\alpha^2)}{40\,t^2}$	
	Short edges fixed Long edges supported	Uniform over entire surface (41)	At centers of short edges (4) $S_a = \frac{0.75\,w b^2}{t^2(1+0.8\,\alpha^4)} = \text{Max. } S$ At center (4) $S_b = -\frac{0.75\,w b^2}{t^2(1+0.8\,\alpha^2+6\,\alpha^4)}$ (4) $S_a = -\frac{0.09\,w b^2(1+3\alpha^2)}{t^2(1+\alpha^4)}$	
	Short edges supported One long edge fixed the other free	Uniform over entire surface (42)	At center of fixed edge (4) $S_b = \frac{3 w b^2}{t^2(1+3.2\,\alpha^3)} = \text{Max. } S$ At center of free edge (4) $S_a = -\frac{0.75\,w a^2}{t^2(1+0.285/\alpha^4)}$	At center of free edge (4) Max $y = -\frac{1.37\,w b^4}{E t^3(1+10\,\alpha^3)}$
Continuous, solid indefinitely large	Supported at equal intervals on circular supports of radius r_0	Uniform over entire surface (43)	At edge of support (8) $S_a = \frac{0.15\,w(a-\frac{4}{3}r_0)(\frac{1}{n}+4)}{t^2}$ when $n > 0.15, < 0.30$ $(n = 2r_0/a)$ or $S_a^{(9)} = \frac{3\,w a^2}{2\pi m t^2}\left[\log\frac{a}{r_0}(m+1) - 21(m-1)\frac{r_0^2}{a^2} - 0.55m - 1.50\right]$ when $n < 0.15$	

(1) Empirical formula based on strain measurements. See "Stresses Produced in a Circular Plate by Eccentric Loading and by a Transverse Couple" R.J. Roark, Bulletin 74, Eng. Exp. Station, Univ. of Wis., 1932
(2) Approximate formula, after Morley. See "Strength of Materials", A. Morley, Longmans, Green & Co., London 1919, p. 441
(3) Approximate formula obtained by interpolation, linear with respect to α, between values for an infinitely long strip and a circular plate
(4) Based on coefficients given by Timoshenko. See "Applied Elasticity", S. Timoshenko and J.M. Lessells, Westinghouse Technical Night School Press, Pittsburgh, 1925
(5) Approximate formula, after Timoshenko. See "Über die Biegung der allseitig unterstützten rechteckigen Platte unter Wirkung einer Einzellast" S. Timoshenko, Der Bauingenieur, Jan. 31, 1922, p. 51
(6) Approximate formula, after Nadai. See "Über die Spannungsverteilung in einer durch eine Einzelkraft belasteten rechteckigen Platte" A. Nadai, Der Bauingenieur, Jan. 15, 1921, p. 11
(7) Approximate formula, obtained by linear interpolation, with respect to α, between values for an infinitely long strip and a square plate
(8) Approximate formula, after Westergaard. See "Moments and Stresses in Slabs", H.M. Westergaard and A. Slater, Proc. American Concrete Inst., Vol. 17, 1921
(9) Approximate formula, after Nadai. See Der Bauingenieur, Vol. V, 1924, p. 1